Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 255 APRIL, 1953 No. 4

GENESIS OF A GENERATOR—THE EARLY HISTORY OF THE MAGNETRON

BY

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SUMMARY

The history of the magnetron is surveyed, starting with its inception in the United States in 1921 as a diode of academic interest. Further development is traced through the intensive investigations in wartime England which culminated in the utilization of the magnetron as a high-power resonant cavity device. Its return to this country in 1940 and the resulting expansion of practical microwave radar is mentioned. The contributions of many early workers in the field are discussed with frequent references to their original papers.

INTRODUCTION

The magnetron is actually one of the oldest of the types of electron tubes which have subsequently found great utility as generators of microwave energy. Magnetron studies both of theoretical and experimental nature began about 20 years before World War II. When the possibilities of the magnetron were first appreciated, workers in the United States, Great Britain, Germany, Czechoslovakia, Japan, France, and Russia began intensively to conduct research to increase understanding of the magnetron art. Later, with the pressure of total war on an England greatly in need of microwave radar, key discoveries began to be made as the cavity resonator, which had proved successful in the development of the klystron, was applied to the magnetron. Then, in the space of a few years, the magnetron was developed as a practical generator of microwave energy.

EARLY MAGNETRONS

In 1921, A. W. Hull (1)[2] initiated magnetron history by a consideration of the trajectories of electrons in a cylindrical diode in the presence

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

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of a magnetic field parallel to the axis of the diode, establishing the Hull cut-off criterion. Hull originally thought of his device as an electronic switch controlled by a magnetic field; later, some experiments were undertaken by him at the General Electric Company's laboratories, utilizing the tube as an amplifier and as an alternating current generator. In 1925, Hull was able to obtain 8 kilowatts output power at 30 kilocycles with an efficiency of 69 per cent. That such a diode could be made to oscillate at a very high frequency, if the electrodes of the tube were made part of a reasonably high-impedance, low-loss resonant circuit, was demonstrated by August Zacek (2), of Prague, in 1924, who operated his tube at 29 centimeters. Independently, Hidetsugu Yagi (3) of the Imperial University at Sendai, Japan, produced microwave energy from a similar tube about this same time. These tubes employed what have come to be known as cyclotron oscillations.

With a greatly increased magnetic field, the diode magnetron, and especially such magnetrons utilizing a split anode, were shown to be capable of a new, more efficient and more reliable form of oscillation of the negative resistance type. Here, the oscillation frequency was determined to a large degree by the natural frequency of the resonant circuit, the magnetic field not being critical as in the case of magnetrons employing cyclotron frequency oscillations. This negative resistance type of operation is often referred to as the Habann-type oscillation. Eric Habann (4) of Jena, in a 1924 theoretical study, proposed that the cylindrical anode of the diode magnetron be cut into two hemi-cylindrical segments as one of several possible electrode systems having negative resistance characteristics. The 1927 experimental work of K. Okabe (5) a co-worker of Yagi, confirmed Habann's considerations and appears to have first attracted general attention to the practical possibilities of the split anode magnetron as a short-wave generator. Okabe was able to receive signals from his 40-centimeter wavelength, split anode tube up to a distance of one kilometer.

A third type of oscillation also obtains in the split anode magnetron and was first investigated experimentally and theoretically by K. Posthumus (6). His development, in 1934, of a four-segment magnetron and the traveling wave oscillation theory of the operation of this tube is of great significance. Of the three possible types of oscillations in magnetrons—cyclotron frequency, negative resistance, and traveling wave oscillations—the last has proved to be the effective mode of operation for practical microwave magnetrons. The important step taken by Posthumus was correctly to interpret the electronic nature of the oscillations in his four-segment tube as an interaction mechanism between the electrons and the tangential component of a traveling wave radio-frequency field, whose velocity is substantially equal to the average velocity of the electrons. Later, F. Herringer and F. Hulster (7) recognized that the radial component of the rotating traveling field

wave has an important role in keeping the electrons in proper phase for efficient interaction with the radio frequency field.

In 1939, E. G. Linder (8) reported a contribution to the magnetron art which showed an important trend toward the designs used in more modern devices. This was a split anode, tank-circuit magnetron in which the split cylindrical anode was made approximately one-quarter wave in length, the two anode segments being connected at one end to act as a tank circuit. An output of 20 watts at 8 centimeters and an efficiency of 22 per cent was achieved.

C. W. Hansell (9), C. W. Rice (10), A. L. Samuel (11) and many others were active in the magnetron field prior to 1940. In general, however, while the results produced by early magnetron research were of considerable academic interest and served to generate a general picture of the mechanism of oscillation of the magnetron, none of the tubes devised was particularly suited to use in practical communication systems. Only with more mature understanding of circuits suitable for microwave-length use, could there follow a significant advance in the magnetron art.

CAVITY RESONATORS AND ELECTRON BEAMS: THE KLYSTRON

With the onset of World War II in Europe in September, 1939, the need for the use of microwave energy in airborne intercept radar sets became apparent in England. A development program was undertaken by M. L. Oliphant and J. Sayers in the Physics Department at the University of Birmingham under a contract with the Department of Scientific Research and Experiment of the Admiralty Office to develop a high power klystron oscillator as a microwave transmitter (12). As was previously understood, the remarkable feature of the klystron was its use of W. W. Hansen's (13) high efficiency closed cavity resonators, which permitted no uncontrolled radiation loss of electromagnetic energy. Resonators of this character had low internal losses, high frequency stability, and, if properly designed, afforded large heat dissipation. The initial British endeavor was to improve the klystron as originally reported as a centimeter wave power generator by R. H. and S. F. Varian (14) in the United States at Stanford University. Just preceding the British research on the klystron, there had been designed at Stanford by John R. Woodyard (15) and W. W. Hansen a demountable klystron producing about 500 watts of 42.5 centimeter continuous wave radiation; this work was made possible through the sponsorship of the Sperry Gyroscope Company. At the Birmingham laboratory, worthwhile results were obtained in extending the operation of the klystron to higher power and to pulsed operation (16). A klystron oscillator producing 400 watts continuously at 7.5 centimeters and which could be pulsed to produce one-thousand-watt peak pulses was achieved. The design of this demountable tube made use of most of

the early Stanford results, with the exception that the resonators were massive copper structures and no cavity grids were used. Actually, this tube was not later employed in radar use, but found considerable application by the British in continuous wave- and pulse-modulated communication systems during the war. Sealed-off versions of this tube, known as the CV-80 and CV-81, produced from 100 to 300 watts continuously in the wavelength range from 7 to 11 centimeters.

These results were promising, and indeed, if circumstances had favored a continuation of the British klystron program, much more satisfactory results would have been obtained. That such might have come about is clearly demonstrated by the recently reported successes at the laboratories of the Sperry Gyroscope Company, where C. Veronda and V. Learned (17) have developed a cascade klystron amplifier employing a space-charge focused electron beam of 4.9 amperes total beam current at 13 kilovolts and producing peak powers of 16,000 watts at 25 centimeters with precise crystal-controlled stability at an efficiency of 25 per cent. In the same laboratory, C. Rich and T. Sege (18) have reported a peak pulsed power output of 40 kilowatts at 3.2 centimeters with a high level power gain of 14 db. in a magnetically focused klystron; a 32-kilovolt 6-ampere electron beam is employed. E. L. Ginzton (19) and his coworkers at Stanford University have also reported very satisfactory cascade klystron amplifier operation with peak pulsed power outputs of 10 megawatts at 10 centimeters at efficiencies of 20 per cent. All of these pulsed klystrons lend themselves to excitation by a crystal oscillator controlled klystron multiplier chain, providing extremely precise frequency regulation.

However, it appears that there were major difficulties in the way of the group at the University of Birmingham in attempting to improve the klystron by a large factor with the techniques then available and under the difficulties of the war period. The primary problem was that of putting enough power into the electron beam of the klystron; this electron beam was of necessity required to have somewhat limited cross-sectional dimensions by the nature of the Hansen resonators, but a more serious problem was the lack of understanding of electron beam focusing systems as recently investigated by C. C. Wang (20), Nelson Wax (21) and A. L. Samuel (22). Before the possibilities of the klystron as a pulse tube were completely explored, the invention of the cavity resonator magnetron occurred. Extensive interest did continue in England and in the United States in the development of the reflex klystron oscillator as a receiver local oscillator source. As discussed by D. R. Hamilton, J. K. Knipp, and J. B. H. Kuper (23), J. R. Pierce (24), A. E. Harrison (25), E. L. Ginzton *et al.* (26), and A. F. Pearce *et al.* (27), this work led to considerable field use of the reflex klystron during the war and subsequently.

THE INVENTION OF THE CAVITY-RESONATOR MAGNETRON

During the early stage of development of the Birmingham klystron, H. A. H. Boot and J. T. Randall (12, 28) were attempting in the same laboratories to develop Barkhausen-Kurz tubes as detectors for use in the proposed airborne microwave radar system. With the objective of building an oscillator for use in laboratory experiments with the Barkhausen-Kurz tube, they turned their attention to the magnetron oscillator as a possible test instrument. Apparently, neither of these physicists had any great previous knowledge or experience in the magnetron art; thus, they were unhampered by prejudices based on the preceding experience of others or on the confused statements in the rather large amount of literature already in existence on magnetrons.

They were, however, familiar with the Hansen and Varian papers and with the results of klystron and other microwave research performed by coworkers in their own laboratory. They also were aware of the desirable properties of Hansen's re-entrant resonator as employed in the klystron; they set about adapting it to use in the magnetron. In Randall's own words:

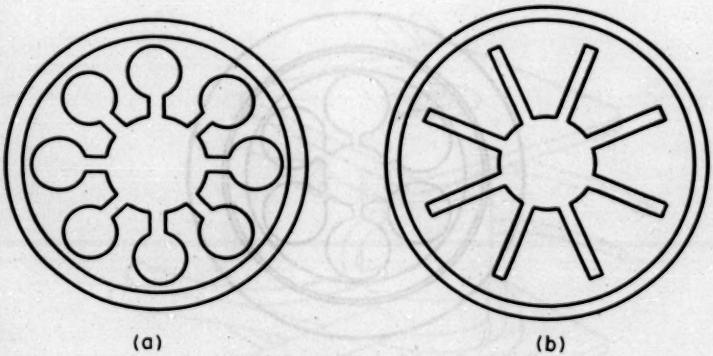

FIG. 1. End view of resonator systems. Type (*a*) is based on a Hertzian dipole while (*b*) is based on a short-circuited quarter-wave line.

> The Hansen papers were by this time available in the laboratory, but it was clear that the designs (hollow spheres, cubes, and "doughnut"-shaped cavities) could not be associated with the cylindrical symmetry of the magnetron. It was decided, therefore, to use either a three-dimensional extension of the well-known Hertzian wire loop or a corresponding extension of a short-circuited quarter-wave line. That either type lends itself to a symmetrical magnetron construction may be seen from the mid-plane sectional drawing (at right angles to the cylindrical axis) of Figs. 1*a* and 1*b*. The general type of design illustrated in Fig. 1*a* was the first to be used in Birmingham, and a drawing of one of the first six anode blocks to be made is given in Fig. 2.

As the entire anode element was machined from a single copper block, the resulting cavities were grouped around a cylindrical cathode placed in the interaction space so that effective heat transfer and high power dissipation might be afforded. The anode block also served as a portion of the vacuum envelope, permitting easy access to the block for direct fluid-cooling purposes, a feature which had been lacking in previous designs and which contributed greatly to its later success. The end regions of the resonators above and below the anode block were defined by parallel metal plates also forming part of the vacuum envelope, thus preventing uncontrolled radiation loss from the anode block.

One must acknowledge that in other laboratories, microwave magnetrons had been under construction making use of anode structures having somewhat similar physical appearance to the British anode, but these devices always lacked some of the important features of the

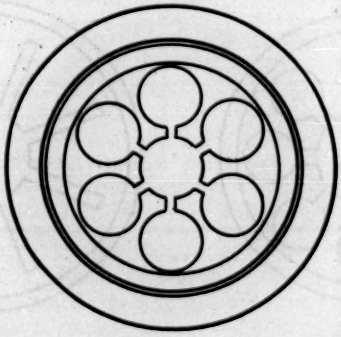

FIG. 2. One of the first six anode blocks made.

British magnetron and were inefficient and incapable of high power output. None of them ever produced more than a fraction of the energy generated by the original Birmingham magnetron. For instance, the U. S. Patent No. 2,063,342 (filed December 8, 1934) to A. L. Samuel is sometimes referred to as the original proposal for the multi-cavity resonator anode; however, the majority of Samuel's important claims were disclaimed and now appear in the Hansell patent 2,217,745 (filed March 20, 1934) and in the Linder patent 2,157,179 (filed July 2, 1934). In both the Samuel and Hansell devices, the resonant circuit consists of grooved anode inductive elements and the capacitance between the cathode and the anode. Also, N. T. Aleskeev and D. E. Malairov (29) tell of Russian work in 1936 on a low performance magnetron with an anode asserted to be related to the British design.

The completely new magnetron structure was designed by Boot and Randall (28) in November, 1939, including the true cavity resonator type of anode system, the number of cavity resonators and their dimensions, and the coaxial line output device, the latter following the design of the coupling loop output circuit previously used in the klystron. As Randall (12) described the new magnetron:

> Six resonators, 1.2 centimeters in diameter, with slots 0.1 × 0.1 centimeter, were employed, and the length of the resonator was 4 centimeters; it was hoped that the wavelength of this system symmetrically surrounding an anode hole of the same diameter would be approximately 10 centimeters. Preliminary calculations had shown that this design might be expected to operate reasonably well with an applied potential of 16,000 volts and a magnetic field of approximately 1,000 oersteds. . . .

A drawing of the first demountable tube, constructed during December, 1939, appears in Fig. 3; it used waxed seals, as facilities for

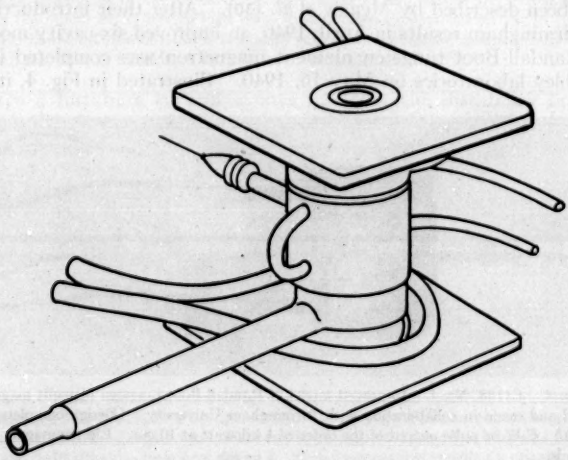

FIG. 3. The first (demountable) cavity magnetron.

making sealed-off tubes were not yet available at the Birmingham laboratory. As no pulsed power supply was at hand, the tube was operated continuously, employing a 0.75-millimeter diameter tungsten wire as a cathode. The tube operated successfully at its first trial on February 21, 1940, radiating about 400 watts of 9.8-centimeter energy with an estimated efficiency between 10 and 15 per cent (28).

At once, a program to extend these results was initiated; it included, among others, the following objectives:

(*a*) The construction of a sealed-off version of the new tube.
(*b*) The construction and test of other designs of anodes, including anodes with different types and numbers of resonators.
(*c*) The introduction of large diameter cathodes in order to increase current density in line with the then recent results of Gutton and Berline of Compagnie Général Télégraphie sans Fils with split anode magnetrons.
(*d*) The use of pulsed operation.
(*e*) The improvement of r-f power measurement and other experimental techniques.

DEVELOPMENT AND INITIAL MANUFACTURE OF THE BRITISH MAGNETRON

The laboratories of the General Electric Company, Limited, at Wembley, were brought into the picture to cooperate in the design of a sealed-off tube for manufacture; their subsequent accomplishments have been described by Megaw *et al.* (30). After their introduction to the Birmingham results in April, 1940, an improved six-cavity model of the Randall-Boot tungsten filament magnetron was completed in the Wembley laboratories on May 16, 1940. Illustrated in Fig. 4, it used

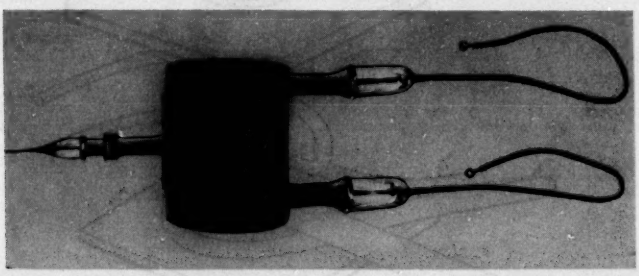

FIG. 4. E1188, No. 1. Improved model of Randall-Boot tungsten filament magnetron; designed and made in collaboration with Birmingham University. (Design completed May 16, 1940.) C-W or pulse output of the order of ½ kilowatt at 10 cm. Electro-magnet weight 50 pounds.

completely closed copper ends for the vacuum envelope, the cathode being supported from a pair of radially extending terminal posts. The performance of this sealed-off version was similar to that of the original Boot-Randall model. The General Electric and Birmingham laboratories introduced large diameter oxide-coated cathodes in test magnetrons in May, 1940, enabling peak power outputs to be extended to from 10 to 50 kilowatts at efficiencies reaching 20 per cent. This proved that the results of Gutton and Berline with large cathodes in split anode magnetrons could be extended to the cavity resonator

magnetron. Samples of the French cathodes had been available to the British government just before the fall of France in June, 1940.

At Birmingham, experiments continued finally to prove that the cavity resonator idea as applied to magnetrons was of a general nature; successful construction and test were made of an 8-cavity, 5-centimeter wavelength anode, a 14-cavity, 5-centimeter tube, a 6-cavity, 3-centimeter tube, and a 30-cavity, 2-centimeter tube, the latter having been tested by the end of September, 1940. These and other 1940 experiments on stability defeated pessimistic views expressed by many established experts whose thinking had apparently been so clouded by the muddled conclusions of the prior magnetron art that they failed to appreciate the mode of operation of the new magnetron oscillator. According to Boot and Randall (**28**):

> In the early stages, it was thought that the resonator magnetron would suffer from the disadvantages of the simpler split-anode type in which the frequency was largely determined by the anode voltage and magnetic field; earlier magnetrons also had the reputation of having high noise values. Experiments carried out in July, 1940, showed that the variation of frequency with anode voltage and magnetic field was small and difficult to measure with the equipment then available.

Perhaps a fortunate turn of events was that the instability brought about by oscillations on different modes did not become troublesome until some time after these initial experiments indicating good stability.

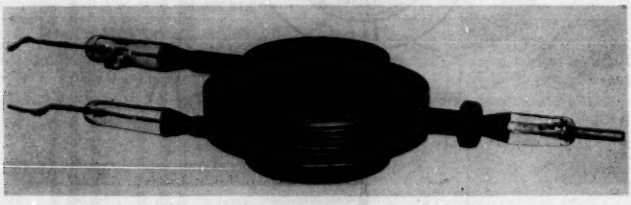

FIG. 5. E1189, No. 1. Original design for high-power pulse operation in aircraft. (Design completed May 25, 1940. First operated June 29, 1940.) Pulse output of the order of 10 kilowatts at 10 cm. (Output with 1000-oersted permanent magnet: 3 kilowatts from first design and 5–10 kilowatts from final design.) Permanent-magnet weight six pounds.

In the General Electric laboratories at Wembley, progress continued toward the design of a prototype tube for manufacture (**30**). This tube, the E1189 (Serial No. 1) of Fig. 5, had as its main features a large cathode, a small axial dimension allowing the use of a minimum air gap for the magnet, and compact air-cooled all-metal construction. A thoriated tungsten spiral cathode was used; in the next (Serial No. 2) tube, an oxide cathode was substituted with improved results. The primary internal dimensions of the E1189 (Serial No. 2) were:

Number of cavities	6
Anode diameter	1.2 cm.
Cathode diameter (emission 5 amperes at 7 volts, 1.8 amperes heater current; for 1 millisecond pulses)	0.45 cm.
Anode length	2.0 cm.
Cavity hole diameter	1.2 cm.
Slot width	0.2 cm.
Slot depth	0.2 cm.

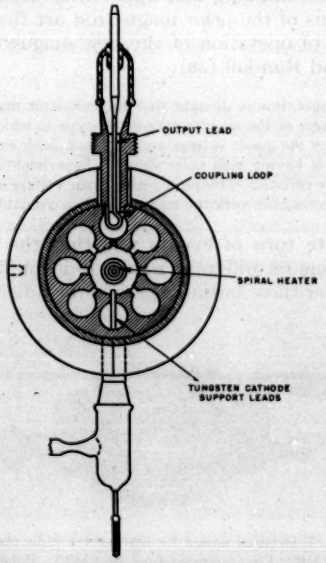

FIG. 6. Internal construction of E1189 magnetron in final 8-segment form (from Serial No. 12). Anode length 2 cm. Anode diameter 1.6 cm. Cathode diameter 0.5 cm.

Both of these 9.8-centimeter wavelength tubes afforded 1-kilowatt peak pulses of 5 to 40 microseconds duration with a 1000-oersted magnetic field on June 29, 1940. With the oxide-coated cathode, output was essentially independent of heater voltage, reaffirming the importance of the role of secondary electron back bombardment of the cathode, which had previously been observed by Boot and Randall and by many others in earlier types of magnetrons. Within two weeks, outputs of 10 kilowatts and higher were obtained with a greater magnetic field (1400 oersteds). The two tubes were in the hands of experimental radar

groups in July, 1940, and other tubes of the same design were immediately constructed.

The design of the E1189 (Serial No. 2) seemed good, except that theory indicated that a lower and more convenient magnetic field might possibly be used with an eight-cavity resonator tube. Figure 6 illustrates the resulting design; this magnetron was later standardized for production for use by the British navy as the NT98. Serial Number 12

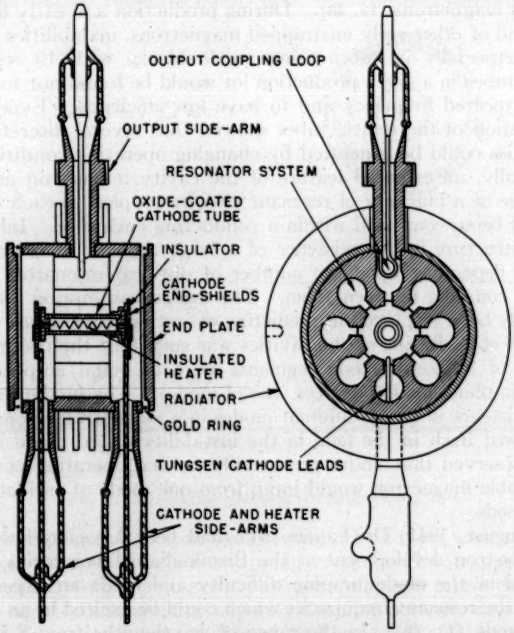

FIG. 7. Magnetron type NT98.

of this design was the first tube disclosed in the United States. The manufacture of the production version (the NT98) of the E1189 appears to have been undertaken in England by the British Thomson-Houston Company and by the Marconi-Osram Valve Company, as well as by the General Electric Company. The tube is illustrated in Fig. 7, and can be seen to resemble closely the E1189; it produced peak pulse power outputs as high as 30 kilowatts. Willshaw *et al.* (31) have described its design and production in detail; by the end of 1941, when improved types began to appear, two thousand NT98's had been manufactured in

England. Willshaw *et al.* (31) also discuss attempts to produce 100- and 150-kilowatt, 8-resonator magnetrons based on direct extensions of the design of the NT98.

THE INVENTION OF THE STRAPPED ANODE AND ITS DEVELOPMENT

Perhaps second only in importance to the basic invention of Boot and Randall was the invention in August, 1941, by J. Sayers in the University of Birmingham laboratories, of the strapped cavity anode block for magnetrons (12, 28). During production and early use of the NT98 and of other early unstrapped magnetrons, instabilities were observed, especially in tubes using anode blocks with 10 resonators. Certain tubes in a given production lot would be found not to oscillate at the expected frequency and to have low efficiency. Experimental investigation of the erratic tubes showed that several discrete output frequencies could be generated by changing operating conditions.

Actually, an essential feature of the cavity magnetron anode lies in the use of a plurality of resonant circuits coupled to each other by virtue of being contained within a conducting enclosure. Inherent in such a structure is the existence of a large number of resonant frequencies depending upon the number of physical resonators and the mode of coupling between them. The design assumption which had previously been employed in predicting operating wavelength was that the mode of excitation of the cavities was such that the instantaneous voltages of adjacent anode segments were of equal amplitude and differed in phase by 180 degrees (the pi mode). The fundamental fact of the existence of many different modes in a magnetron resonator system showed itself in the field in the instability called mode jumping. It was observed that under presumably normal operating conditions, the unstable magnetron would jump from one mode of oscillation to a nearby mode.

In August, 1941, Dr. Sayers, who had been associated with high power klystron development at the Birmingham laboratories, became interested in the mode-jumping difficulty and made an experimental study of the resonant frequencies which could be excited in an 8-cavity E1189 anode (12, 28). In the range of wavelengths from 8.5 to 10.5 centimeters, three resonances were found and a small probe was used to study the corresponding voltages on the faces of the anode segments. One of these modes was of large amplitude and was determined to be the desired pi mode; in the other two modes, alternate anode segments had zero amplitude voltage. Sayers reasoned that the latter two modes might be eliminated by connecting together one set of alternate segments by appropriate conductors at one end of the anode and by similarly connecting the intervening set of alternate segments at the opposite end of the anode. When this was done, only the desired pi mode was observed in the wavelength region mentioned previously.

The double-ring and echelon-strapping methods and others were promptly tested. In double-ring strapping, two narrow concentric grooves are turned near the inner ends of the anode segments at each end of the anode, and castellated copper rings, made by bending into a circle strips shaped as in Fig. 8*a*, are inserted in each groove. The distance between castellations being twice the distance between anode segments, alternate segments are coupled together. With echelon strapping (Fig. 8*b*), small U-shaped copper wires are fastened into holes in

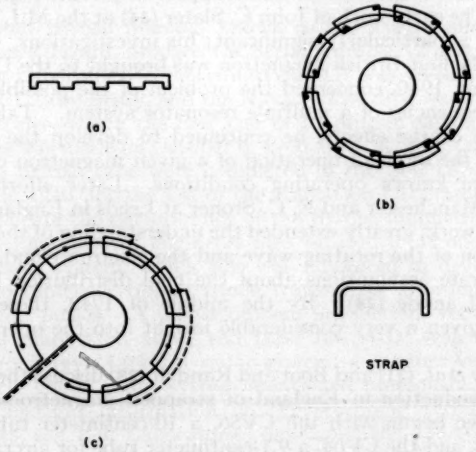

FIG. 8. Illustrating strapping of magnetron segments.

the anode segments. The first scheme tested by Sayers, which has sometimes been referred to as Y-B strappings, was the wire strapping system of Fig. 8*c* in a 10-cavity magnetron; no straps appear across the cathode support region, thus avoiding capacity coupling between the straps and the cathode supports. As described by Boot and Randall (28):

> From five samples an average efficiency of 40 per cent was obtained. The lowest efficiency obtained was 33 per cent and the highest 50 per cent. The average efficiency of a number of unstrapped valves of this type (10 cavity) was below 10 per cent and such valves were probably not working in the π mode. When (straps were) applied to the E1189 type of valve, efficiencies up to 65 per cent were obtained giving peak outputs of over 150 kw, whereas the highest efficiencies obtained with this type valve unstrapped were 40 to 45 per cent.
>
> These increases in output power were perhaps the most striking outcome of the use of strapping, but the most important effect was the almost complete freedom from mode jumping. . . . The increase in power is really a secondary effect of strapping for, due to the removal of other modes, it is possible to increase the anode current and voltage without coming too near to the operating conditions for another mode.

The function of strapping is twofold. It separates the resonant frequencies of the modes widely, so as to avoid overlapping of the desired operating mode by undesired modes. Also, it distorts the field patterns of the undesired modes to such a degree that oscillations cannot be coupled to the electrons by these modes, leaving the desired pi mode with a regular wave pattern. Complete theoretical discussions of the role played by strapping and of the electronic mode of operation of the strapped and unstrapped magnetron have been recorded by Collins (32) and Fisk *et al.* (33), and appear in private papers of W. W. Hansen. The early work of John C. Slater (34) at the M.I.T. Radiation Laboratory is particularly significant; his investigations, which began soon after the first British magnetron was brought to the United States in September, 1940, concerned the problem of the possible number of resonant frequencies of a multiple resonator system. Taking into account space charge effects, he continued to develop the theory to a degree that the mode of operation of a given magnetron could be calculated from known operating conditions. Later efforts of D. R. Hartree at Manchester and E. C. Stoner at Leeds in England, based on Dr. Slater's work, greatly extended the understanding of the mechanism of interaction of the rotating wave and the electron cloud, using more nearly accurate assumptions about the field distribution between the cathode and anode (28). By the middle of 1942, these theoretical efforts had given a very considerable insight into the operation of the magnetron.

Willshaw *et al.* (31) and Boot and Randall (28) discuss the subsequent successful production in England of strapped magnetrons, which appears to have begun with the CV56, a 10-centimeter tube for naval transmitters, and the CV64, a 9.1-centimeter tube for aircraft interception equipment. The CV56 had a stable efficiency of 40 per cent. The CV64 appears to have been the first production echelon-strapped tube, and the tube was pretuned during construction to the design frequency by deforming the straps suitably. Many thousands of these tubes were manufactured by the British Thomson-Houston Company beginning early in 1942, after a considerable number had been produced by a magnetron production unit attached to the Birmingham laboratories.

DISCLOSURE IN THE UNITED STATES

Early in September, 1940, a technical mission came to the United States to divulge progress in England toward the development of microwave radar (32, 33). On the basis of facts brought to this country at the time, a British-American agreement of cooperation was soon formulated, and the National Defense Research Committee, organized in the middle of 1940, began at once to lay plans for immediate extension of the development of microwave radar in the United States. Thus began a continuous flow of information from the British magnetron and

other research centers to the N.D.R.C. and particularly to the Radiation Laboratory at the Massachusetts Institute of Technology after its organization by the N.D.R.C. in November, 1940, under Dr. L. A. DuBridge. In DuBridge's (35) own words:

> . . . the Radiation Laboratory became a focus of the great international cooperative endeavor in the field of microwaves. The cooperation of the British . . . continued with ever-increasing effectiveness throughout the war. O.S.R.D. and the Radiation Laboratory gathered information from British agencies and distributed it to all American agencies, and vice versa.

The original British mission, known as the Tizard mission for its leader Sir Henry Tizard, brought the Serial Number 12 magnetron from the Wembley Laboratory to this country and it was revealed to officials of the N.D.R.C., the Army, and the Navy. The tube was loaned to the Bell Laboratories, where the first test of its output capabilities in the United States was made on October 6, 1940, by J. B. Fisk (33). Several copies of the British tube were completed in the Bell Laboratories by November 15, 1940, for the use of the Radiation Laboratory and were found to duplicate the performance of the British magnetron.

The Bell Laboratories engineers also utilized it at once in a 700-megacycle radar system, which was well advanced in development at that time, to replace a pulsed triode transmitter affording only two-kilowatt peak pulses. A non-strapped production version of this experimental magnetron tube, the type 700 A-D, was used in the United States Navy Mark III radar system, a product version of the 700-megacycle triode radar system mentioned previously (36). After the British brought information on the strapping technique to the United States, the Bell Laboratories developed the 728A-J magnetron using, in the production prototype, the original British Y-B wire strapping, and, in the production model, double-ring strapping.

CONCLUSION

It is not the purpose of these notes exhaustively to review the literature which discusses the contributions made by industry and by universities in the United States to the magnetron art since 1940, when the basic information on the British cavity resonator magnetron was first disclosed here. This has been well covered by others especially by Collins (32) and by Fisk, Hagstrum, and Hartman (33). More specific contributions from the Radiation Laboratory, Columbia University, Cornell University, and the Bartol Research Foundation are recorded elsewhere in the literature, which also shows important contributions to the art by the Bell Telephone Laboratories, Westinghouse, General Electric, and the Radio Corporation of America. Willshaw *et al.* (31) describe the concurrent work done in England to extend operation of magnetrons to shorter wavelengths and to higher powers, to improve cathodes, strapping and tuning methods, and power output transmission

line systems, to develop packaged magnetrons in which the magnetic circuit is integral with the tube, and to develop c-w magnetrons. As many agencies had become involved in magnetron research by the end of 1942, it is difficult, indeed, to trace from the literature the true origin of various discoveries and inventions.

The purpose of these notes is rather to call particular attention to the basic contributions to this art made by British research workers, especially those by Randall, Boot, and Sayers, who were awarded a prize of 36,000 pounds by the British Royal Commission on Awards to Inventors for the original development which they carried on in the laboratories of the University of Birmingham (37). It can be stated safely that no single contribution changed so profoundly the course of the battle against the submarine menace during World War II. Further, these notes would be incomplete without the remark that the accomplishments of the British physicists were based upon and inspired by the discoveries of W. W. Hansen, who, in the words of Dr. John C. Slater (38), more than anyone else deserves to be considered the founder of microwave electronics.

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Lightweight Triangulation Tower.—A lightweight, air-transportable triangulation tower which can be erected at heights varying from 37 to 103 ft. in less than one working day by an experienced five-man crew has been developed by the Army's Engineer Research and Development Laboratories, Fort Belvoir, Va.

Made of aluminum, its structural members weigh only 3617 lb.—3020 lb. less than the standard World War II steel tower of the same height. Complete with two sets of anchors, platforms, extra bolts and a chest for small parts, the entire ensemble weighs 4152 lb.

Dismantled, the new tower can be transported easily in a standard Army 2½-ton truck and can be dropped from a helicopter to troops in inaccessible areas without suffering extensive damage.

The new structure, whose longest member is 14 ft. and heaviest member but 60 lb., is actually two mutually independent towers in one. The inner tripod is fitted with an instrument table while the outer one (113 ft. high) serves as a working platform for the observer and his recorder, and as a support for signal lamps used as targets.

The tripods are completely separated to prevent the surveyor's movements from disturbing the delicate adjustments of the instrument on the inner tower.

Developed for use in military surveys, the tower has undergone stability, rigidity and air-drop tests to meet military requirements.

Aqualung.—A little-known saga of "Engineering in Exploration" is set forth in an article bearing that name by Dr. Gilbert Grosvenor, President of the National Geographic Society, in a recent issue of *The American Engineer*, official magazine of the National Society of Professional Engineers.

In this article, Dr. Grosvenor predicts that an engineering device invented especially for exploration will have wide application. It is the aqualung, a strange new device which permits men to swim free and unfettered hundreds of feet beneath the ocean.

"Looking like something described in a science-fiction novel, the aqualung will be used on a four-year, round-the-world expedition sponsored by the National Geographic Society to study life in most of the world's oceans. The device was perfected by Explorer Captain Jacques-Yves Cousteau with the aid of a brilliant young Parisian Engineer, Emile Gagnan, of Air. Liquide Cie, France's largest commercial gas firm.

"A comparatively simple device, the aqualung consists essentially of three tubes of compressed air mounted on the diver's back with intake and exhaust tubes leading to a mouthpiece. There is a special regulator valve which automatically feeds compressed air to the mouthpiece in ratio to the diver's depth.

"Until the development of the aqualung, divers were limited in the depths to which they could descend by the tremendous depths of the ocean. Now, however, the deeper the diver goes, within limits, the higher the pressure of the compressed air forced into his body by the aqualung. The special regulator valve keeps the pressure inside the diver's body equal to the pressure outside. He feels no more discomfort under enormous pressures than does a fish."

ON THE FUNDAMENTALS OF THE BOUNDARY LAYER THEORY

BY

M. Z. KRZYWOBLOCKI [1]

INTRODUCTION

As is well known, the engineering sciences were created by means of some combination of more fundamental scientific fields such as mathematics, physics, chemistry, etc., with the special purpose of enabling one to apply in a very direct way the fundamental results to highly practical problems of immediate use. At the beginning of the existence of those sciences, an engineer used to follow more or less precisely the paths of reasoning of such a fundamental field like, say, mathematics. But in the course of years, his way of reasoning began to deviate more and more from the rigorous logic of mathematics. Being anxious to supply an immediate result, he began to use his initiative and sense of invention to create various methods of solutions of differential and integral systems without proving the existence and uniqueness theorems at all. Sometimes these proofs were derived by pure mathematicians after the engineer proposed the method, as was the case with the Heaviside calculus. Sometimes the proofs presented by an engineer did not make any sense, as was the case with the so-called Principle of Least Work Method in the theory of elasticity. In the 20th century the situation became still worse, due to the fact that the number of such "engineering" solutions increased beyond human imagination. The situation became paradoxical, particularly when the large engineering circles with an inadequate mathematical background believe in preposterous proofs cited in various text and hand-books written by other engineers of a similar theoretical background. A good example is the so-called Principle of Least Work Method, where rigorous mathematical proofs do not succeed in convincing engineers that the method has weak mathematical justification and is only a random choice method. It may be that the engineer is unable to follow the mathematical derivations and stubbornly preserves his tradition simply because no other better method is available. On the other hand, a mathematician wonders why the word "mathematics" is abused by an engineer, why it is not preserved only for the field whose logics and derivations follow the principles of mathematics in the fully rigorous meaning of this word and why some other name was not invented for those manipulations consisting of some simple operations like differentiation, integration,

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etc., which are used by an engineer without any rigorous justification, having the only confirmation in form of physical experiment.

Below, the author will discuss the fundamentals of the boundary layer theory. In particular, he will show that there does not exist any rigorous mathematical theory of the boundary layer due to the lack of solution of the corresponding boundary value problem of non-linear partial differential equations. A discussion on the fundamental equations underlaying the boundary layer theory in hypersonic flow will close the present note.

1. PRANDTL'S PROPOSITION

Prandtl[2] (1904) proposed and Blasius (1908) worked out the first approximate solution of the boundary layer equation along a flat plate situated horizontally for an incompressible fluid with a constant coefficient of viscosity. The fundamental Prandtl's assumptions based upon physical observations were the following: (*a*) the boundary layer thickness is very small (which is true at moderate velocities in a high density medium): (*b*) as a consequence of (*a*) the vertical velocity component in the boundary layer is negligible and may be assumed to be almost equal to zero; (*c*) as a consequence of (*a*) and (*b*) the gradient of the pressure in the vertical direction is almost equal to zero; (*d*) as a consequence of (*a*), (*b*) and (*c*) the resultant velocity in the potential flow outside the boundary layer is horizontal and equal to the uniform velocity of the incoming flow. Prandtl put forward the hypothesis of a certain "metric" of the boundary layer in an incompressible flow in form of the thickness of the boundary layer δ proportional to $Re^{-\frac{1}{2}}$, with Re denoting the Reynolds Number. These assumptions allowed Prandtl to reduce the number of equations from three to two and to reduce the number of terms in one of the two remaining equations. From physical tests it was noticed that the velocity distribution in the boundary layer is similar to a parabola. This suggested to Prandtl the use of a transformation function $z = \text{Const.}\, yx^{-\frac{1}{2}}$, which function transforms the system of the reduced non-linear partial differential equations onto a system of ordinary differential equations in one independent variable z, solvable by the use of power series expansion, say. But neither Blasius nor anybody else after him did prove the existence and the uniqueness of the boundary value problem in question in a rigorous way.

2. DEVELOPMENT OF SOLUTIONS OF THE BOUNDARY LAYER

Prandtl-Blasius solution was and is followed by numerous methods of solutions of the boundary layer problem. All of them are formal procedures without going into the details of the existence and uniqueness of the actual boundary value problem. In the realm of a compressible fluid one may distinguish the following groups of methods of solution:

[2] For this and other literature references in the paper, see the list appended to this paper.

(*a*) transformations that work under suitable simplifying assumptions about the parameters μ (viscosity), ρ (density) and κ (heat conductivity). One may include here the methods of von Mises, Crocco, Hantsche-Wendt, Howarth and others; (*b*) transformations that adapt the compressible boundary layer equations to the application of approximate methods developed for the incompressible boundary layer (Pohlhausen, etc.): (*c*) transformations that prepare the equations for a numerical computer. One may include here series expansions, mostly power series which automatically imply the analyticity of the functions in question, propositions by Howarth, Hartree, Emmons and Brainerd, asymptotic solutions, etc. Concerning the asymptotic solutions the authors do not present the consideration on the order of the boundedness of the remainder which item mathematically is the most important since it may indicate how large is the discrepancy between the asymptotic solution and the actual value of the function in question. In all the methods mentioned above the existence of the solution is taken for granted.

3. BOUNDARY VALUE PROBLEM

Let us discuss the possibilities of proving the existence and uniqueness of the boundary value problem of the boundary layer. Assume that the heat factor is also taken into account, that is, the equations of state and energy are included. In this case the most general system of the boundary layer equations represents a system of non-linear higher order partial differential equations with (at least) two-curve boundary conditions (inner—at the surface of the body—and outer boundary curves).

As is known, the problem arising with such a differential system is to find, from the entire class of solutions of the differential system that solution or those solutions which satisfy certain additional conditions. Since problems of this type arise from physical considerations, certain properties are expected of the solution: (i) existence of a solution; over-sufficient conditions should not be applied as they may be incompatible and no solution will be found. Existence of a solution may not always be assured by physical intuition; (ii) uniqueness of the solution; sufficient conditions should be provided to determine the solution uniquely; (iii) the solution should depend continuously on the data; this is necessary for a physical problem as the boundary and boundary values can be ascertained only within a small latitude of variation (the so-called "natural" problems). One may consider two possibilities:

(i) The possibility of solving the boundary value problem as a problem from the theory of partial differential equations. Let us discuss briefly these possibilities. As is known, the boundary conditions by means of which one can determine a unique solution of the partial differential equation of the second order are of different character, depending upon the type of equation: elliptic, hyperbolic or parabolic.

The classical theorem of Cauchy-Kowalewsky attempts to introduce a certain uniformity in the approach to the solution of such equations provided all the functions in question are analytic. But this uniformity in the approach ceases to be valid when the hypothesis of analyticity does not hold in the problem in question. Let us exclude from our consideration the parabolic type as being completely exceptional. Regarding the elliptic type the data may generally be expressed in one of three standard forms: (*a*) on a closed boundary S the value of the function u is specified (the boundary value problem of the first kind or that of Dirichlet); (*b*) on a closed boundary S the value of $\partial u/\partial n$ (or any of the first partial derivatives) is specified (the boundary value problem of the second kind or that of Neumann); (*c*) "mixed" boundary value problem, that is, choosing u on one part of S and its first partial derivative on another part of S. In some problems one can consider a certain linear combination of the two above data.

The second order partial differential equations of the "normal" hyperbolic type fulfill exactly the theorem of Cauchy-Kowalewsky. On an open curve (initial curve) which may be finite, the values of u and $\partial u/\partial n$ are specified. This is Cauchy problem. Generally, there are two curves joining the initial curve, going up to infinity, on which either u or $\partial u/\partial n$ are given (boundary curves). Hadamard mentioned that for the non-normal hyperbolic type no solution of the type mentioned above ("correctly set") is known. It is of importance to notice that in general, the elliptic type of equation usually does not admit the given data in form of Cauchy which are fully admissible for the hyperbolic type. Inversely, in the latter case it is of disadvantage to assume a closed curve which is convenient for the elliptic type.

The higher order equations "totally" elliptic (all the characteristic directions are imaginary) usually involve the following data: u and $\partial u/\partial n$ are specified along a closed curve or on a closed surface (Hilbert, E. E. Levi, Fredholm). The "totally" hyperbolic equations (all the characteristic directions are real) have been solved by Holmgren (by the Riemann method). The "composite" type of equations (characteristic directions partly real, partly imaginary) has been attacked by Hadamard $\left(\frac{\partial}{\partial x}\Delta u = \Delta\frac{\partial u}{\partial x} = 0, \frac{\partial^4 u}{\partial x^4} - \frac{\partial^4 u}{\partial y^4} = 0\right)$, Sjöstrand, Davis $(b_1 u_{xxx} + b_2 u_{xxy} + b_3 u_{xyy} + \cdots = 0)$, etc. The data are usually formulated in the following way: Let T be a closed subset of R, with a rectifiable boundary W. Let $W' \subset T$ be a curve extending from W to W: given u on W and $\frac{\partial^2 u}{\partial x^2}$ on W', or u on W and W', or u_x on W and u on W', etc. Although such an equation is neither of pure elliptic nor hyperbolic type, one is compelled to assume that the equation should possess the properties of an elliptic character, which implies that one of the curves

must be closed. To systems of first order equations one may refer the problem of Cauchy or initial value problem (one curve).

From this brief discussion it seems obvious that one cannot expect a solution of boundary layer problem in this group, that is, when the boundary conditions are formulated in the usual way on two curves: (*a*) $y = 0$, $u = v = 0$; (*b*) $y = \infty$ or $y = \delta$, $u = U_0$, $v = \partial u/\partial y = 0$. The boundary curves intersect at an angle, of course.

(ii) The possibility of a transformation of partial differential system into an ordinary one will be discussed next. For the sake of the simplicity we restrict our consideration to two independent variables. Assume a function $W = f(x,y)$ in two independent variables $\{x, y\}$ subject to a formal transformation into $\{u, v\}$ independent variables. Following the principles of classical analysis one obtains:

$$W = f(x, y) = F(u, v); \quad J = J\left(\frac{\phi, \psi}{u, v}\right); \quad x = \phi(u, v); \quad y = \psi(u, v);$$

$$u = g(x, y); \quad v = h(x, y); \quad f_{,x} = \begin{vmatrix} W_{,u} W_{,v} \\ \psi_{,u} \psi_{,v} \end{vmatrix} J^{-1}; \quad f_{,y} = -\begin{vmatrix} W_{,u} W_{,v} \\ \phi_{,u} \phi_{,v} \end{vmatrix} J^{-1}.$$

Moreover, the following relations will be used:

$$dy = d\psi = \psi_{,u}du + \psi_{,v}dv; \quad du = dg = g_{,x}dx + g_{,y}dy;$$
$$dv = dh = h_{,x}dx + h_{,y}dy.$$

Introducing the latter two equations into that one for dy gives the expression of the form: $dy = (\cdots)dx + (\cdots)dy$. Since the variations dx and dy are independent and arbitrarily large, the first coefficient must vanish and the second must be equal to one. This gives two relations:

$$\psi_{,u}g_{,x} + \psi_{,v}h_{,x} = 0; \quad \psi_{,u}g_{,y} + \psi_{,v}h_{,y} = 1.$$

As the first condition we require that $u = \phi = x$, which implies

$$\phi_{,u} = 1; \quad \phi_{,v} = 0; \quad J = \phi_{,u}\psi_{,v} = \psi_{,v}; \quad g_{,x} = 1; \quad g_{,y} = 0;$$
$$f_{,x} = W_{,u} - W_{,v}\psi_{,u}/\psi_{,v} = W_{,u} + W_{,v}h_{,x}; \quad f_{,y} = W_{,v}/\psi_{,v} = W_{,v}h_{,y}.$$

As the second condition we require that $W = F(v)$ only, which implies $u = 0$ or $=$ const., $W_{,u} \equiv 0$, $x = \phi = 0$ or $=$ const., $\psi_{,u} = 0$. One gets: $f_{,x} = \dfrac{dW}{dv}h_{,x}$; $f_{,y} = \dfrac{dW}{dv}h_{,y}$. But the Jacobian $\left(J\dfrac{(\phi, \psi)}{u, v}\right)$ vanishes identically and the transformation is not one-to-one. Let $v \equiv z = y/f(x)$, f being arbitrary. Thus the functions $f_i(x, y)(i = 1, 2, \cdots, n, \cdots)$, satisfying a given system of non-linear partial differential equations $\theta_i(x, y, f_i, f_{i,x}, \cdots) = 0(i = 1, 2, \cdots)$, are transformed onto the functions $F_i(z, x)$ belonging to a certain class (C), satisfying a system of ordinary

differential equations $\Gamma_i = 0$, by means of some transformation function $z = z(x, y)$, fulfilling the conditions mentioned above. The transformation from partial derivatives in the (x, y, f_i)-space into ordinary derivatives in the $(x', z, F_i(z, x'))$-space, with $x' \equiv x$, is the main feature of this procedure. In the transformed system there appear only ordinary derivatives with respect to z. This means that x in $\Gamma_i = 0$ must be a parameter and also that the transformation is not one-to-one. As mentioned above, in the case of partial differential system one deals with a two-curve boundary value problem. In the case of an ordinary differential system one has a two-point boundary value problem. As is known, certain results were obtained in problems of this kind.

(iii) The boundary conditions will be discussed next. In the regime of an incompressible fluid or in the regime of a compressible fluid at low Mach Numbers the Prandtl's postulates can be accepted to be valid. This means that the boundary conditions on both boundary curves are well defined and given, the boundary layer is relatively very thin and the vertical pressure gradient is equal to zero. However, in the regime of a hypersonic flow in a rarefied gas the situation is different. One may distinguish here two sub-regimes: (*a*) hypersonic flow in a rarefied gas in continuum and (*b*) in slip flow. In the first one the inner boundary conditions may be assumed to be given, that is, the velocity is zero at the surface of the body. But the boundary layer is thick, sometimes very thick, in which case the vertical velocity component and the vertical pressure gradient inside the boundary layer are no longer of a negligible magnitude and definitely must be taken into account. This is the reason that the number of equations cannot be reduced. The vertical velocity component produced by the viscous flow at the outer edge of the boundary layer itself generates the external pressure field. The boundary conditions at the outer edge of the boundary layer are not given in the form of a definite number, but in form of a function which depends upon the (unknown at the beginning) solution of the non-linear boundary layer equations. Such a type of a "non-linear" (as one may call it) boundary value problem appears more and more often in fluid mechanics. But, as is well-known, there is at present no solution to that problem and the only possibility is to apply some approximative iteration approach. In one of his papers, the author suggested a method of successive approximations. The situation complicates in a slip flow. In this case the inner boundary conditions are also "non-linear." For example, the slip velocity at the surface of the body is given by the expression $u|_{y=0} = C(u_{,y})_{y=0}$. The picture complicates almost beyond any hope if one includes the shock phenomena at the leading edge of the body. There may be required the boundary conditions of one type at the surface of the body (possibly non-linear), of another type behind the leading-edge shock (these may be given by the phenomena existing behind the shock) and still of another type (maybe non-

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linear) on the outer boundary curve. The outer boundary curve may not coincide with the shock curve, it may only intersect it.

A rigorous solution of the boundary value problem of the boundary layer phenomenon is the first fundamental problem.

4. FUNDAMENTAL EQUATIONS OF THE BOUNDARY LAYER

In general, one may distinguish two main regimes in the flow of a gas: Clausius regime, $K = d\lambda^{-1}$ being large (the symbol d denotes a characteristic dimension of the body and λ the free path) and Knudsen regime, K being small. Notice that there are authors who define the number K as equal to λd^{-1}. One may include in the first regime the mechanics of continua (the fluid sticks to the surface of the body) and the slip flow; the second regime may refer to the region of free molecule flow (Newtonian flow). The region of a rarefied gas may be contained partly in both regimes. Further, in both regimes the free path is assumed to be of such a length that the binary collisions are sufficient to describe adequately the phenomena in question. Of course, the question remains open whether the binary collisions are always sufficient; the phenomena at the front of the body in motion, near the stagnation point, may require a revision of that assumption.

As mentioned above, Prandtl and Blasius proposed a certain approximate solution of the boundary layer problem along a flat plate in an incompressible fluid. In the course of years their approach was extended to include curved surfaces, temperature pattern and compressible fluids. All those approaches were fundamentally based upon the assumption that the Navier-Stokes equations of motion (momentum) and energy (derived by means of concepts of the mechanics of continua) are adequate enough to describe the phenomena in question. But recently the flights at very high altitudes have been considered, that is, in hypersonics and in the range of a rarefied gas, or in other words, in the upper part of the Clausius regime, where both types of flow-continuum and slip flow may occur. In connection with that the question arises whether the Navier-Stokes equations are adequate enough to describe accurately the phenomena in that range. Let us discuss briefly this question.

The only other possibility is to refer to the kinetic theory of gases. As is well-known, the Maxwell-Boltzmann equation refers to a monatomic gas. A particular solution of this equation was proposed in works of Hilbert, Enskog, Chapman and others, and is known under the name of Burnett's equation. A certain generalization of Burnett's equation to polyatomic gases (beginning with the Maxwell-Boltzmann equation) was proposed by C. S. Wang-Chang and G. E. Uhlenbeck. They derived only the first order terms for a polyatomic gas. As is known, the derivation of higher order terms in Burnett's proposition requires a great amount of time and energy.

These approaches preserve always the physical notions of the coefficients of viscosity and heat conductivity, as they are used today. There are various objections against Burnett's equations. Truesdell proposed a purely formal series expansion of the continuous function of several variables expressing the values of the shearing stress tensor and heat flux vector. The coefficients in these expansions are gradients of gradients. Hence they are new in physics and are unknown for the time being, unless some physical experiments in the future will be performed to determine them. Grad recently proposed another particular solution of the Maxwell-Boltzmann equation by use of Hermite polynomials. This is considered today to be the best method. But, in general, due to the lack of the general theory of non-linear equations one may be unable to investigate the influence of various parameters and assumptions upon the final form of the solution in question. Hence it may be difficult, today, to decide in an absolute manner which particular solution of the Maxwell-Boltzmann equation describes in the best way the physical phenomenon of the boundary layer, say, in the hypersonic regime, unless a great number of particular examples is solved by means of high speed computing devices, so that the general theoretical trend of the considered form of solution is visible. The question may become more complicated when other physical phenomena than those included in the solutions discussed above, will be taken into account. The possible very high temperature gradient in the boundary layer, which occurs in the flow in a hypersonic regime, jointly with the high speed of flow may originate such phenomena as ionization, dissociation, condensation and difference in electromagnetic charges. Moreover, the relaxation phenomena (establishment of equilibrium between the translational and the internal energy states when the former is changed by some process, for example, by a collision) should be necessarily included. Concerning the appearance of the dissociation phenomenon, the opinion is divided, although there are some who say that it does not take place in the boundary layer. The recent investigations at Cornell University proved that there appear significant electromagnetic phenomena in shocks due to a difference in electric charges. As is known, the phenomenon of a shock resembles closely the phenomenon of the boundary layer (steep variation in the velocity, temperature, and density distribution) and as Ludford recently had shown, the equations of a shock are in their certain phases, analogous to the equations of the boundary layer. The above phenomena are predominantly completely ignored in the existing forms of equations of the boundary layer. Some considerations on the subject of the dissociation phenomenon were presented by L. L. Moore. Some consideration of the subject of the relaxation time are contained in the British A.R.C.R. and M. Report by Gunn and in the report by C. S. Wang-Chang and G. E. Uhlenbeck, mentioned above. A certain initiative in the direction

of considering the electromagnetic phenomena was undertaken by B. E. Howard in his thesis (University of Illinois), although the fundamental equation assumed is that of Navier-Stokes, in which a few new terms are added.

The Knudsen regime is, as yet, more or less in the sphere of speculations. The usual assumption is that the Newton's law of resistance is valid and that the physical picture in question is the model proposed by Newton: the particles hit the surface of the body and bounce backward.

The next question is the form of the equation of state. As is known, the basic hypotheses of the kinetic theory of gases as ordinarily presented restrict the validity of that theory to ideal gases ($p = R\rho T$). The question arises, of course, whether the status of the real gas in all the realms in question can be adequately described by the equation of state in the form $p = R\rho T$, $R = \text{const.}$

To all the controversial items discussed above, one more may be added. In 1951 Slezkin and Vallender (USSR Academy of Sciences) published two papers in which they attacked the ordinary, well-known, equation of continuity. Their reasoning is as follows: In the usual continuity equation one takes into account the density changes due only to the microscopic velocities but not to diffusion, whereas in treating the energy transfer both the convective heat transfer and the molecular phenomenon of heat conduction are considered. In their opinion the equation of continuity should be revised and after doing that they obtained new forms of equations of continuity, motion (momentum) and energy. One item must be strongly emphasized, namely, that the present form of the continuity equation as well as all the other equations of motion and energy can be derived from the Maxwell-Boltzmann equation. Slezkin and Vallender based their approach upon the usual assumptions of the mechanics of continua. Recently Šapošnikov criticized this hypothesis of Slezkin and Vallender and concluded that it does not correspond to a physical reality.

The establishment of definite, fundamental equations of motion and energy valid in the boundary layer, based upon sound physical assumptions, containing all the physical phenomena, discussed above, which are considered to be of a certain importance for the distribution of the velocity, temperature and density in the boundary layer, is the second fundamental problem of the boundary layer phenomenon of today.

FINAL REMARKS

The author discussed briefly above the two fundamental problems of the boundary layer phenomenon: the existence and uniqueness theorems and the fundamental equations governing the velocity, temperature and density distributions in the boundary layer. It seems that the existence theorem can be derived by means of the available methods of mathematics of today, but that the same cannot be said

concerning the uniqueness theorem. Hence one is unable to state whether or not the boundary conditions as usually given by the physical principles are sufficient to determine the solution uniquely. Regarding the fundamental equations governing the velocity and temperature distributions, the opinions are divided and no conclusions can be reached without a considerable number of numerical examples to be calculated by means of high-speed computing devices.

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New Axle Tester.—A unique axle-testing machine, capable of simulating in a laboratory actual driving conditions that a truck weighing 80,000 lb. and traveling at a speed of 60 mph. would experience, was installed recently in the Alden Indoor Proving Ground of the Timken-Detroit Axle Company in Detroit, Mich.

The testing equipment, engineered and built by the General Electric Company, duplicates torques, stresses, and speeds that the drive axle of a truck or bus would encounter under both normal and strenuous loads.

The new axle-tester was shown publicly for the first time at the annual meeting of the Society of Automotive Engineers in Detroit on January 12. An audience of 1,000 automotive engineers, their guests and military personnel, seated in Rackham Educational Memorial, viewed an exclusive closed-circuit telecast showing the axle-testing equipment in operation at the Timken-Detroit laboratory, five miles away. A description of the various parts and operations of the testing machine was televised from the laboratory to the floor of the SAE convention.

Incorporating huge dynamometers, the largest ever used for the purpose, and intricate electronic equipment to control their action, this machine can impose tests on axles never before possible indoors.

A G-E 350-hp. dynamometer simulates the truck engine driving the axle through transmission gears, and can deliver up to 450 hp. over a drive shaft speed range of from 400 to 3000 rpm. Integral blowers are provided for cooling to allow motoring speeds down to creeping speeds. Two output dynamometers, rated 550 hp., 650–3000 rpm., are attached to gears located at the ends of the axle and—by means of measuring devices—record the torque output. The combined measurements of the three dynamometers test the efficiency of the axle. The gears, located at the ends of the axle, allow wide variations of speed and load.

Amplidyne equipment provides field power for the three dynamometers and a 750-kw. motor-generator set. Only one MG set is required because of special loop circuits installed in the system.

G-E also supplied the operator's control console for the system and a unique cycling control which contains two strip chart recorders for measuring speed and torque.

Truck and bus axles are tested by placing them on the test stand and subjecting them to highway conditions by means of a magnetic tape. This tape, when run through the automatic cycling equipment, causes the dynamometers to respond to the impulses on the tape and exert the indicated speeds and stresses on the axle. The tape recorder provides repetition of the opertor's speed and torque signals used in an initial manual test. As each stress is imposed, recording instruments indicate continuously the speed, torque, or load as well as provide a permanent record. The same test can be repeated many times until the durability or life expectancy of the axle is established. The recording tape can be filed, and the test repeated exactly at any later date.

Safety measures, consistent with dynamometer practice, are provided which offer protection both to operating personnel and equipment. These include overload and overspeed devices and dead-front enclosed floor-mounted cubicles for all control.

So far, the axle-testing machine has been used for life tests, endurance runs, and shock loading tests. Other tests are being developed.

THE ALGEBRAIC THEORY OF LINEAR TRANSMISSION NETWORKS *

BY

MAYNARD G. ARSOVE [1]

Part I [2]

1. INTRODUCTION

Transmission networks are (as the name implies) a means of transmitting information in the form of impulses of electrical energy from one place to another. The information is supplied to the transmission network through a preferred set of junction points, called "input terminals," and is collected from a similar set of "output terminals" at the receiving end. We consider here networks of this type with the further restriction that there be an equal number of input and output terminals.

Such networks constitute a semi-group under the operation of series combination (that is, interconnection of the output terminals of one network with the corresponding input terminals of another), and it is this concept that forms the basis of our theory. Of particular interest is the set of those networks which are reversible in the sense that information can be *sent* from the output terminals and *received* at the input terminals, since this set of networks is a group.

Adopting the semi-group point of view, we obtain a number of fundamental properties of transmission networks, as well as a classification of some of the principal groups of networks. In particular, with the aid of a factorization theorem (theorem 2) we are able to derive a simple criterion (theorem 3) for determining when a transmission network behaves as though constructed of symmetrical impedance elements. Questions relating to the characteristic and iterative impedances are also investigated, one of the main results being a necessary and sufficient condition for the existence of a characteristic impedance (theorem 14). This theorem, which does not seem to be currently available even for the 4-terminal case, invokes for its proof the theory of analytic functions of matrices.

Finally, we show how the theory of continuous semi-groups yields a concise definition (definition 19) and rigorous treatment of the general (n-wire) transmission line. The continuous semi-group approach does

* Research conducted while the author was recipient of a National Research Council Predoctoral Fellowship.

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[2] Part II will appear in this JOURNAL for May, 1953.

not require the assumption of section symmetry of the transmission line, and it is of interest to note how much of the theory can be developed without this assumption. Moreover, the infinitesimal transformations associated with a given transmission line are useful in determining properties of the line.

2. PRELIMINARIES

In what follows we consider only *linear* networks, and we shall be concerned with network *analysis*, rather than synthesis. Since our viewpoint is primarily algebraic, it is convenient to by-pass the topological problems by starting with the loop equations as characterizing a given linear network.

Definition 1. By a (*linear*) *network* we mean a linear transformation $e = Z\,i$, where e and i are k-dimensional column vectors and Z is a $k \times k$ matrix, all having complex numbers as elements. We term e the *emf vector*, i the *current vector*, and Z the *impedance matrix* of the network.

To solve certain theoretical problems, we shall occasionally find it desirable to "construct" special types of networks. It should be emphasized, however, that these "constructions" are in the sense of definition 1 (that is, purely mathematical) and that questions of actual physical realizability are beyond the scope of this paper. Nonetheless, as in the case of the factorization theorem, mathematical constructions of this sort may lead to results applicable to the *analysis* of physical networks.

In what follows we agree to work entirely within the complex field, so that the elements of all vectors and matrices with which we deal will be (as stated explicitly in definition 1) complex numbers. The k-dimensional unit matrix will be denoted throughout by U_k, and the determinant of a square matrix A by $\det A$. A further notational convention which we adopt is the use of an asterisk to indicate the transpose of a matrix. Thus, the scalar product of e and i appears as e^*i or i^*e.

Definition 2. A network $e = Z\,i$ will be called *symmetrical* provided its impedance matrix is a symmetrical matrix, that is $Z^* = Z$.

3. FIXED-TERMINAL NETWORKS AND THEIR TRANSFORMATIONS

For many applications one employs physical networks in which certain of the nodes are set aside as *terminals* and interest is centered on the emf's and currents as measured at these terminals. Control networks, as well as transmission networks, fall into this category. Denoting the p-dimensional column vectors giving the terminal emf's and currents by e_t and i_t, respectively, we can write the loop equations in block form as

$$\begin{pmatrix} e_t + \epsilon_t \\ \epsilon \end{pmatrix} = \begin{pmatrix} Z_t & \xi \\ \eta & \Psi \end{pmatrix} \begin{pmatrix} i_t \\ \iota \end{pmatrix},$$

where ϵ_t, ϵ, and ι are column vectors of dimensions p, $k - p$, and $k - p$, respectively, which represent internal emf's and currents. This leads in a natural way to

Definition 3. By a (*linear*) *fixed-terminal network* of *index* p we mean an affine transformation of the form

$$\begin{pmatrix} e_t \\ 0 \end{pmatrix} = \begin{pmatrix} Z_t & \xi \\ \eta & \Psi \end{pmatrix} \begin{pmatrix} i_t \\ \iota \end{pmatrix} - \begin{pmatrix} e_t \\ \epsilon \end{pmatrix},$$

where e_t, i_t, e_t are p-dimensional column vectors, ϵ and ι are $(k - p)$-dimensional column vectors, and the submatrices Z_t, ξ, η, and Ψ have the respective dimensions $p \times p$, $p \times (k - p)$, $(k - p) \times p$, and $(k - p) \times (k - p)$. We call e_t the *terminal emf vector* and i_t the *terminal current vector.*

If n is the number of terminals of the given physical network, then $n \geq p + 1$. In many (but not all) cases the equality will hold, depending on the topological structure of the given physical network. We carry over the notion of impedance matrix and that of network symmetry to the fixed-terminal case.

Definition 4. The fixed-terminal network of definition 3 will be called *passive* provided

$$\begin{pmatrix} e_t \\ \epsilon \end{pmatrix} = 0.$$

(Otherwise, *active*).

Any fixed-terminal network of index p determines a class of fixed-terminal networks externally identical to it in the following sense.

Definition 5. Two fixed-terminal networks N and N' of index p will be called *externally identical* provided the terminal emf and current vectors e_t and i_t satisfying the conditions imposed by the affine transformation N are exactly those satisfying the conditions imposed by N'.

We examine now the class $K(N)$ of those fixed-terminal networks N' externally identical to a given fixed-terminal network N which are generated by the linear transformations

$$\begin{pmatrix} e_t' \\ 0 \end{pmatrix} = \begin{pmatrix} U_p & x \\ 0 & X \end{pmatrix} \begin{pmatrix} e_t \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} i_t' \\ \iota' \end{pmatrix} = \begin{pmatrix} U_p & 0 \\ y & Y \end{pmatrix} \begin{pmatrix} i_t \\ \iota \end{pmatrix},$$

where p is the index of N, x and y are matrices of dimensions $p \times (k - p)$ and $(k - p) \times p$, respectively, and X and Y are non-singular $(k - p) \times (k - p)$ matrices. Expressing the affine transformations in full matrix form as

$$\begin{aligned} N: &\quad e = Zi - \mathcal{E} \\ N': &\quad e' - Z'i' - \mathcal{E}' \end{aligned}$$

and introducing the notation

$$S = \begin{pmatrix} U_p & x \\ 0 & X \end{pmatrix}, \qquad T = \begin{pmatrix} U_p & 0 \\ y & Y \end{pmatrix}, \tag{1}$$

we have $e' = Se$ and $i' = Ti$. It follows without difficulty that the transformations induced on Z and ϵ are

$$\begin{cases} Z' = SZT^{-1} \\ \mathcal{E}' = S\mathcal{E}. \end{cases} \tag{2}$$

Since S is non-singular, we see that *the fixed-terminal networks of $K(N)$ are either all passive or all active.* Also, although the externally supplied power must be invariant, *the internally supplied power is generally variant* in view of the fact that $\mathcal{E}^*i$ is transformed into $\mathcal{E}^*S^*Ti$.[3] Taking $T = (S^*)^{-1}$ yields a tensor transformation under which Z is a doubly covariant tensor: $Z' = SZS^*$. *Transformations of this type maintain invariant the internally supplied power and preserve network symmetry.* In fact, with regard to the latter assertion it is clear that $Z^* = Z$ implies

$$(Z')^* = (SZS^*)^* = SZ^*S^* = SZS^* = Z'.$$

We introduce now two purely algebraic notions of "resonance," which characterize certain types of network singularities corresponding to non-dissipative resonance in the physical case.

Definition 6. A network $e = Z\,i$ (or a fixed-terminal network $e = Z\,i - \mathcal{E}$) is said to be in *series resonance* provided Z is a singular matrix.

Definition 7. A fixed-terminal network

$$\begin{pmatrix} e_1 \\ 0 \end{pmatrix} = \begin{pmatrix} Z_1 & \xi \\ \eta & \Psi \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} - \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

is said to be in *parallel resonance* provided the matrix Ψ is singular.

The terminology here is motivated by the simplest physical cases of series and parallel resonance, illustrated in Fig. 1. In connection with

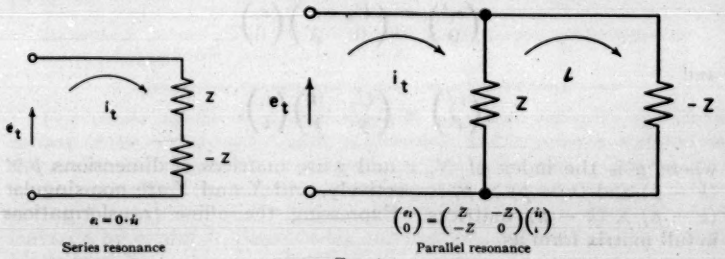


FIG. 1.

[3] Physical considerations suggest the terminology *instantaneous power* for the scalar product e^*i and *average power* for its real part $R\,(e^*i)$, for a given network $e = Z\,i$.

definition 7 we observe that a fixed-terminal network is in parallel resonance when and only when the internal network formed by leaving all terminals unconnected (so that $i_t = 0$) is in series resonance.

Let us now write the transformation (2) on the impedance matrix in terms of the submatrices employed in definition 3, to obtain

$$\begin{cases} Z_t' = Z_t + x\eta - \xi Y^{-1}y - x\Psi Y^{-1}y \\ \xi' = \xi Y^{-1} + x\Psi Y^{-1} \\ \eta' = X\eta - X\Psi Y^{-1}y \\ \Psi' = X\Psi Y^{-1}. \end{cases} \tag{3}$$

From the equations

$$\begin{cases} \det Z' = (\det S)(\det Z)(\det T)^{-1} = [(\det X)/(\det Y)] \det Z \\ \det \Psi' = [(\det X)/(\det Y)] \det \Psi \end{cases} \tag{4}$$

it follows that *all fixed-terminal networks in $K(N)$ have the same status with regard to series (parallel) resonance.* Moreover, when N is not in parallel resonance, we can specify S and T by setting

$$X = Y = U_{k-p}, \quad x = -\xi\Psi^{-1}, \quad y = Y\Psi^{-1}\eta, \tag{5}$$

which results in $\xi' = \eta' = 0$.

In this case the affine transformation N' decomposes as a direct sum, and from this it is clear that N is externally identical to the fixed-terminal network

$$e_t = (Z_t - \xi\Psi^{-1}\eta)i_t - (\epsilon_t - \xi\Psi^{-1}\epsilon) \tag{6}$$

of p loops.

Definition 8. By a *canonical form* of a fixed-terminal network N we mean a fixed-terminal network N_c externally identical to N and having the property that the parameters p and k (of definition 3) for N_c are equal. If such a network N_c exists, then N is said to *admit a reduction to canonical form.*

There follows

Theorem 1. Let N be a fixed-terminal network. If N is not in parallel resonance, then N admits a reduction to canonical form, the canonical form N_c being determined uniquely as the affine transformation $e_t = Z_c i_t - \mathcal{E}_c$, where (in the notation of definition 3 for N) we have

$$Z_c = Z_t - \xi\Psi^{-1}\eta,$$
$$\mathcal{E}_c = \epsilon_t - \xi\Psi^{-1}\epsilon.$$

The canonical form N_c has the properties (1) of being symmetrical whenever N is symmetrical and (2) of being in series resonance if and only if N is in series resonance.

Proof. The reduction to canonical form is obvious from the work leading up to Eq. 6, and the uniqueness of N_c follows immediately from its definition.

If N is symmetrical, then $Z^*{}_t = Z_t$, $\xi^* = \eta$, and $\Psi^* = \Psi$. Hence,

$$Z_c{}^* = Z_t{}^* - \eta^*(\Psi^*)^{-1}\xi^* = Z_t - \xi\Psi^{-1}\eta = Z_c,$$

showing that N_c is also symmetrical. Finally, the series resonance property follows from Eqs. 4, since here

$$Z' = \begin{pmatrix} Z_c & 0 \\ 0 & \Psi' \end{pmatrix}$$

and

$$\det Z_c = (\det Z')/(\det \Psi') = (\det Z)/(\det \Psi).$$

An elementary example of reduction to canonical form is afforded by the bridge network of Fig. 2. The loop equations yield the im-

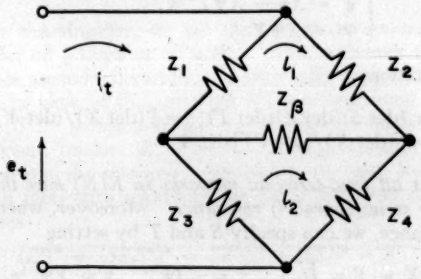


FIG. 2.

pedance matrix

$$Z = \begin{pmatrix} Z_1 + Z_3 & -Z_1 & -Z_3 \\ -Z_1 & Z_1 + Z_2 + Z_\beta & -Z_\beta \\ -Z_3 & -Z_\beta & Z_3 + Z_4 + Z_\beta \end{pmatrix}.$$

Whenever the network is not in parallel resonance, the matrix

$$\Psi = \begin{pmatrix} Z_1 + Z_2 + Z_\beta & -Z_\beta \\ -Z_\beta & Z_3 + Z_4 + Z_\beta \end{pmatrix}$$

admits an inverse, and we compute Z_c as

$$Z_c = Z_t - \xi\Psi^{-1}\eta = \frac{Z_\beta(Z_1 + Z_3)(Z_2 + Z_4) + Z_1Z_3(Z_2 + Z_4) + Z_2Z_4(Z_1 + Z_3)}{Z_\beta(Z_1 + Z_2 + Z_3 + Z_4) + (Z_1 + Z_2)(Z_3 + Z_4)}.$$

4. TRANSMISSION NETWORKS AND THE SEMI-GROUP PROPERTY

As remarked in the introduction, we shall be concerned here with the mathematical analysis of linear physical networks having two separate sets of terminals, equal in number, those at the sending end being called the *input terminals* and those at the receiving end being called the *output terminals*. Schematically, this can be depicted as in

Fig. 3, where we indicate also our conventions as to the assumed directions of the input and output emf's and currents.[4]

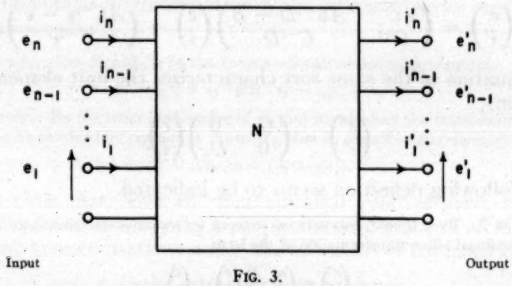


FIG. 3.

Series combination of two such networks N_1 and N_2 is illustrated in Fig. 4. That this combination gives rise to a semi-group is obvious,

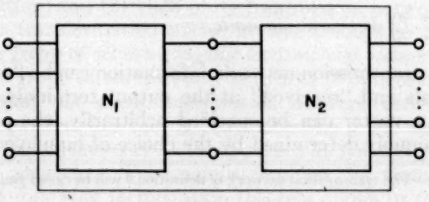


FIG. 4.

since we have (1) closure (the resulting network is linear and has the same terminal arrangement) and (2) associativity. Furthermore, the semi-group has a unit element, which is simply the network consisting of a direct connection between corresponding input and output terminals.

Let us suppose now that the fixed-terminal network of Fig. 3 admits a canonical representation as

$$\begin{pmatrix} e' \\ e \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} i' \\ -i \end{pmatrix} - \begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix}, \tag{7}$$

where e is the input emf vector, i the input current vector, ϵ the input internal emf vector, and the primes denote corresponding output

[4] For the sake of definiteness we indicate (here and in what follows) only one input and one output reference terminal. Of course, whenever the network graph consists of more than one component, more than one input and one output reference terminal will be required.

vectors.[5] Of the square submatrices A, B, C, D we shall require further that C be non-singular, so that Eq. 7 can be rewritten as

$$\begin{pmatrix} e' \\ i' \end{pmatrix} = \begin{pmatrix} AC^{-1} & AC^{-1}D - B \\ C^{-1} & C^{-1}D \end{pmatrix} \begin{pmatrix} e \\ i \end{pmatrix} + \begin{pmatrix} AC^{-1}\epsilon - \epsilon' \\ C^{-1}\epsilon \end{pmatrix}. \tag{8}$$

An equation of the same sort characterizes the unit element (direct connection),

$$\begin{pmatrix} e' \\ i' \end{pmatrix} = \begin{pmatrix} U_n & 0 \\ 0 & U_n \end{pmatrix} \begin{pmatrix} e \\ i \end{pmatrix},$$

and the following definition seems to be indicated.

Definition 9. By a (*linear*) *transmission network* (or *transmission section*) we mean a $2n$-dimensional affine transformation of the form

$$\begin{pmatrix} e' \\ i' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} e \\ i \end{pmatrix} + \begin{pmatrix} \epsilon \\ \iota \end{pmatrix},$$

where $e, i, e', i', \epsilon, \iota$ are n-dimensional column vectors and $\alpha, \beta, \gamma, \delta$ are $n \times n$ matrices. We call e the *input emf vector*, i the *input current vector*, e' the *output emf vector*, and i' the *output current vector*. Further, we term $\begin{pmatrix} e \\ i \end{pmatrix}$ the *input vector*, $\begin{pmatrix} e' \\ i' \end{pmatrix}$ the *output vector*, and $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$ the *transmission matrix*.

Given any transmission network, information can be "sent" from the input terminals and "received" at the output terminals, in the sense that the input vector can be specified arbitrarily, the output vector then being uniquely determined by the choice of input vector.[6]

Definition 10. The transmission network of definition 9 will be called *passive* provided

$$\begin{pmatrix} \epsilon \\ \iota \end{pmatrix} = 0.$$

(Otherwise, *active*.)

Formalizing the relationship between the affine transformations (7) and (8) yields

Definition 11. Let

$$\begin{pmatrix} e' \\ i' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} e \\ i \end{pmatrix} + \begin{pmatrix} \epsilon \\ \iota \end{pmatrix}$$

be a transmission network. If the $n \times n$ matrix γ is non-singular, then we shall say that the given transmission network *corresponds* to the canonical fixed-terminal network

$$\begin{pmatrix} e' \\ e \end{pmatrix} = \begin{pmatrix} \alpha\gamma^{-1} & \alpha\gamma^{-1}\delta - \beta \\ \gamma^{-1} & \gamma^{-1}\delta \end{pmatrix} \begin{pmatrix} i' \\ -i \end{pmatrix} - \begin{pmatrix} \alpha\gamma^{-1}\iota - \epsilon \\ \gamma^{-1}\iota \end{pmatrix}.$$

---

[5] The appearance of $-i$ in Eq. 7 is due to the assumption of current direction as toward the terminals when the physical network of Fig. 3 is considered as a fixed-terminal network, rather than a transmission network. (Note definitions 9 and 11, appearing below.)

[6] In actual physical applications the general situation is that of the output terminals connected in some fashion to a receiving network, so that only the input emf vector can be arbitrarily specified. However, this in no way conflicts with the preceding remark or detracts from the appropriateness of definition 9.

Comparison with Eq. 8 shows this correspondence to be of a reciprocal nature.

A mathematical characterization of the process of series combination (Fig. 4) is immediate.

Definition 12. Let N_1 and N_2 be the $2n$-dimensional transmission networks.

$$t_1' = \chi_1 t_1 + \tau_1 \quad \text{and} \quad t_2' = \chi_2 t_2 + \tau_2,$$

respectively. By the *series combination* of N_1 and N_2 we mean the transmission network $N_1 \cdot N_2$ obtained by composing N_2 on N_1, that is, the affine transformation

$$t_2' = (\chi_2\chi_1)t_1 + (\tau_2 + \chi_2\tau_1).$$

It is thus clear that, from the mathematical point of view, the semi-group of transmission networks is simply the semi-group of all $2n$-dimensional affine transformations under the operation of composition.

5. GROUPS OF TRANSMISSION NETWORKS

We adopt the convention that *in what follows, all the transmission networks considered are of fixed dimension* $2n$. Clearly, a set of transmission networks will form a group if and only if (1) it is closed under series combination, and (2) each of its transmission networks is a non-singular affine transformation whose inverse belongs to the set. *We denote by* Γ *the group of all non-singular transmission networks*, so that Γ is the maximal group of transmission networks of dimension $2n$.

The fact that not every transmission network corresponds to a fixed-terminal network is somewhat disconcerting if one wishes to carry over to transmission networks a concept (such as symmetry) defined for fixed-terminal networks. Fortunately, however, for transmission networks in Γ the following factorization theorem allows us to circumvent this difficulty.

Theorem 2. Every transmission network N in Γ (that is, every non-singular transmission network) can be expressed as a series combination $N = N_1 \cdot N_2$ of two transmission networks N_1 and N_2 having the following properties:

(1) both N_1 and N_2 correspond to canonical fixed-terminal networks, that corresponding to N_2 being symmetrical and passive;

(2) N_2 has a transmission matrix of the form

$$\begin{pmatrix} U_n & 0 \\ \gamma_2 & U_n \end{pmatrix}$$

with $\gamma_2^* = \gamma_2$.

Proof. Let the transmission matrix of N be

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}.$$

The non-singularity of χ then assures us that the $2n \times n$ submatrix $\begin{pmatrix} \alpha \\ \gamma \end{pmatrix}$ has rank n. If γ has exactly m linearly independent rows, then by elementary row operations we can make these the last m rows and also make the remaining $n - m$ rows all zero. This can be expressed as

$$R\gamma = \begin{pmatrix} 0 \\ \gamma' \end{pmatrix},$$

where R is the matrix of these row operations and γ' is the matrix of the last m rows.

Since R is non-singular, the matrix

$$\begin{pmatrix} (R^{-1})^* & 0 \\ 0 & R \end{pmatrix}\begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} (R^{-1})^*\alpha \\ R\gamma \end{pmatrix}$$

likewise has rank n. Hence, among the rows of $(R^{-1})^*\alpha$ there are $n - m$ which are linearly independent and which, together with the rows of γ', form a linearly independent set. By simply interchanging rows of $(R^{-1})^*\alpha$ these $n - m$ rows can be made the first $n - m$ rows, yielding in matrix notation

$$P(R^{-1})^*\alpha = \begin{pmatrix} \alpha' \\ \alpha'' \end{pmatrix}.$$

No row need be interchanged more than once, so that P can be taken as a *symmetrical* permutation matrix.

Now, the linear dependence of the rows of α'' on those of α' and γ' allows us to find matrices q and Q such that

$$\alpha'' = q\alpha' + Q\gamma'.$$

For c any constant there results

$$P(R^{-1})^*\alpha + cR\gamma = \begin{pmatrix} U_{n-m} & 0 \\ q & Q + cI \end{pmatrix}\begin{pmatrix} \alpha' \\ \gamma' \end{pmatrix}.$$

We next particularize c as any non-zero number distinct from the negatives of the m characteristic roots of Q and observe that the two matrices in the right-hand member are non-singular. Hence, the matrix $\gamma_1 = (1/c)R^{-1}P(R^{-1})^*\alpha + \gamma$ is non-singular, and we have

$$\gamma = \gamma_2\alpha + \gamma_1,$$

where γ_2 is the non-singular symmetrical matrix $(-1/c)R^{-1}P(R^{-1})^*$.

This construction enables us to factor χ as

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} U_n & 0 \\ \gamma_2 & U_n \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma_1 & \delta - \gamma_2\beta \end{pmatrix},$$

which we write as $\chi = \chi_2\chi_1$, and the theorem follows by taking N_1 and N_2 as the affine transformations $t_1' = \chi_1 t_1 + \chi_2^{-1}\tau$ and $t_2' = \chi_2 t_2$, respectively.

The resulting factorization is, of course, far from unique.

Perhaps the most important subgroup of Γ is that consisting of all *passive* non-singular transmission networks. For these transmission networks the affine transformations reduce to linear transformations, so that the resulting group is simply the full linear group of dimension $2n$. Combining this observation with the factorization theorem yields a correlation between the concepts of passive transmission network (definition 10) and passive fixed-terminal network (definition 4): (1) if a non-singular transmission network N is passive, then each of the transmission networks in any factorization $N_1 \cdot N_2$ according to theorem 2 corresponds to a passive fixed-terminal network; conversely, (2) if for some such factorization both N_1 and N_2 correspond to passive fixed-terminal networks, then N is a passive transmission network.

Another important subgroup arises from considerations of impedance symmetry.

Definition 13. A non-singular transmission network N will be called *impedance-symmetric* provided that in some representation of N according to theorem 2 the fixed-terminal network corresponding to N_1 (as well as that corresponding to N_2) is symmetrical.

There follows

Theorem 3. A necessary and sufficient condition for a transmission network N to be impedance-symmetric is that its transmission matrix χ satisfy

$$\chi^* S \chi = S, \tag{9}$$

where S is the skew symmetric matrix

$$S = \begin{pmatrix} 0 & -U_n \\ U_n & 0 \end{pmatrix}.$$

In terms of the submatrices[7] of χ this condition is equivalent to the relations

$$\alpha^*\gamma = \gamma^*\alpha, \tag{a}$$
$$\gamma\delta^* = \delta\gamma^*, \tag{b}$$
$$\delta^*\alpha - \beta^*\gamma = U_n. \tag{c}$$
$$\delta^*\beta = \beta^*\delta, \tag{d}$$
$$\beta\alpha^* = \alpha\beta^*, \tag{e}$$
$$\alpha\delta^* - \beta\gamma^* = U_n, \tag{f}$$

Remark. Equation 9 is equivalent to

$$\chi S \chi^* = S, \tag{10}$$

and Eqs. a, c, d are equivalent to b, e, f. Moreover, when γ is non-singular, Eqs. a, b, c imply d, e, f.

Proof. We first establish the equivalence between (9) and (10) by noting that they can be written, respectively, as

$$S^{-1}\chi^* S = \chi^{-1} \tag{9'}$$

and

$$S\chi^* S^{-1} = \chi^{-1}, \tag{10'}$$

and that $S^{-1} = -S$. That Eqs. a, c, d are equivalent to b, e, f then follows from the fact that they are the submatrix versions of (9) and (10), respectively. Finally, a direct calculation shows that when γ^{-1} exists, Eqs. a, b, c imply d, e, f.

Now, if N corresponds to a canonical fixed-terminal network, it is obvious from definition 11 that symmetry of this network is characterized by the equations

$$(\alpha\gamma^{-1})^* = \alpha\gamma^{-1}, \tag{a'}$$
$$(\gamma^{-1}\delta)^* = \gamma^{-1}\delta, \tag{b'}$$
$$(\gamma^{-1})^* = \alpha\gamma^{-1}\delta - \beta, \tag{c'}$$

which are clearly equivalent to a, b, c. But, the latter imply a through f and hence (9).

Returning to the case of arbitrary N, we note that if N is impedance-symmetric, it can be factored according to theorem 2 as $N_1 \cdot N_2$, where N_1 and N_2 correspond to symmetrical canonical fixed-terminal networks. The above considerations then ensure that the corresponding transmission matrices χ_1 and χ_2 satisfy (9). There follows

$$\chi^* S \chi = \chi_1^* \chi_2^* S \chi_2 \chi_1 = \chi_1^* S \chi_1 = S,$$

proving that (9) is *necessary* for N to be impedance-symmetric.

For the converse we suppose that χ satisfies (9), and use (9') to infer its non-singularity. This permits application of theorem 2 to obtain the factorization $N = N_1 \cdot N_2$, where N_2 corresponds to a symmetrical canonical fixed-terminal network. Then χ_2 satisfies (9), and it follows that χ_2^{-1} and therefore $\chi_1 (= \chi_2^{-1}\chi)$ must also satisfy (9). From this we see that N_1, as well as N_2, corresponds to a symmetrical

[7] Here, and in what follows, we adhere to the notation of definition 9 for the submatrices of χ.

canonical fixed-terminal network. Hence, the condition (9) is *sufficient* for N to be impedance-symmetric.

To evaluate the determinants of certain of the transmission matrices that arise in our discussion, we shall have use for

Lemma 1. Let

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

be a transmission matrix in which the submatrix γ is non-singular, and let

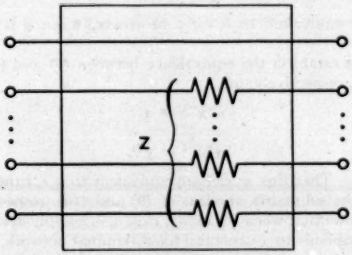
$$Z = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

be the corresponding impedance matrix (in the sense of definition 11). Then

$$\det \chi = (\det \gamma) \det (\alpha\gamma^{-1}\delta - \beta) = (\det B)/(\det C).$$

Proof. Immediate from the factorization of χ as

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma^{-1} & -U_n \\ U_n & 0 \end{pmatrix} \begin{pmatrix} \gamma & 0 \\ 0 & \alpha\gamma^{-1}\delta - \beta \end{pmatrix} \begin{pmatrix} U_n & \gamma^{-1}\delta \\ 0 & U_n \end{pmatrix}.$$



$$\chi = \begin{pmatrix} U_n & -Z \\ 0 & U_n \end{pmatrix}$$

FIG. 5.

In conjunction with theorem 3 this yields

Theorem 4. The set of all impedance-symmetric transmission networks forms a unimodular subgroup of Γ.

Proof. It is easily seen (as noted in the proof of theorem 3) that if condition (9) holds for transmission matrices χ, χ_1, and χ_2, then χ is non-singular and (9) also holds for $\chi_2\chi\chi_1$ and for χ^{-1}. The impedance-symmetric transmission networks therefore constitute a subgroup of Γ.

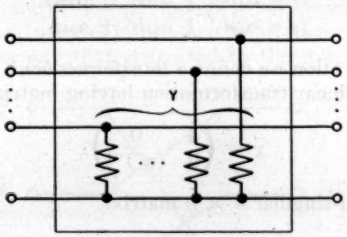
To show this subgroup unimodular, we must show that the matrix χ of any impedance-symmetric transmission network N has determinant one. If N corresponds to a symmetrical canonical fixed-terminal network, then (in the notation of lemma 1)

$$\det \chi = (\det B)/(\det C) = (\det C^*)/(\det C) = 1.$$

The general case is a direct consequence of this, since χ then factors into two matrices each of which has determinant one.

Corollary 4.1. A transmission network N corresponding to a canonical fixed-terminal network N_c is impedance-symmetric if and only if N_c is symmetrical.

The *series impedance subgroup* of Γ is indicated schematically, in terms of a typical element, in Fig. 5. In turn, a subgroup of this subgroup is the *series self impedance subgroup*, obtained by taking Z as a diagonal matrix. Similarly, we have the *parallel impedance subgroup*[8] (Fig. 6) and the *parallel self impedance subgroup* of this obtained by

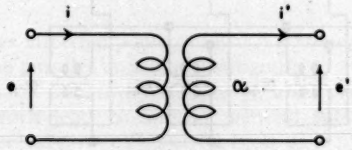


$$X = \begin{pmatrix} U_n & 0 \\ -Y & U_n \end{pmatrix}$$

FIG. 6.

taking Y as a diagonal matrix. All of these subgroups are obviously unimodular.

An (ideal) transformer of ratio $\alpha(\neq 0)$ is a passive four-terminal transmission network of the type indicated in Fig. 7. This can be



$$e' = \alpha e$$
$$i' = (1/\alpha) i$$

FIG. 7.

generalized to higher dimensions in a variety of ways, but the generalization upon which we focus our attention is that depicted in Fig. 8 for the case of $n = 3$. An interconnection of transformers in this special way will be called a "transformer bank." It is evident that the terminal

[8] The transmission network N_3 of theorem 2 belongs to this subgroup.

emf's and currents are related by the equations

$$\begin{aligned} e_1' &= \alpha_{11}e_1 + \alpha_{12}e_2 + \alpha_{13}e_3 \\ e_2' &= \alpha_{21}e_1 + \alpha_{22}e_2 + \alpha_{23}e_3 \\ e_3' &= \alpha_{31}e_1 + \alpha_{32}e_2 + \alpha_{33}e_3 \end{aligned}$$

and

$$\begin{aligned} i_1 &= \alpha_{11}i_1' + \alpha_{21}i_2' + \alpha_{31}i_3' \\ i_2 &= \alpha_{12}i_1' + \alpha_{22}i_2' + \alpha_{32}i_3' \\ i_3 &= \alpha_{13}i_1' + \alpha_{23}i_2' + \alpha_{33}i_3'. \end{aligned}$$

With this as motivation we define a *transformer bank* mathematically as a $2n$-dimensional linear transformation having matrix

$$\chi = \begin{pmatrix} \alpha & 0 \\ 0 & (\alpha^*)^{-1} \end{pmatrix},$$

where α is any non-singular $n \times n$ matrix.

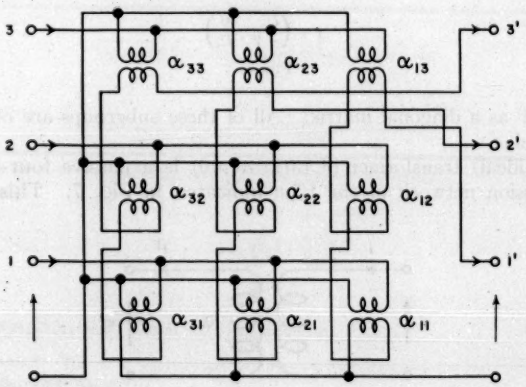


FIG. 8.

From theorem 3 we see that *every transformer bank is impedance-symmetric*, and this accounts for our interest in the particular type of interconnection of transformers shown in Fig. 8. Furthermore, we observe that *the set of all transformer banks forms a unimodular subgroup of* Γ. Taking α to be a permutation matrix π yields a subgroup isomorphic to a group of permutations of output terminals of the direct connection (Fig. 9).[9]

[9] Since π is orthogonal, $(\pi^*)^{-1} = \pi$.

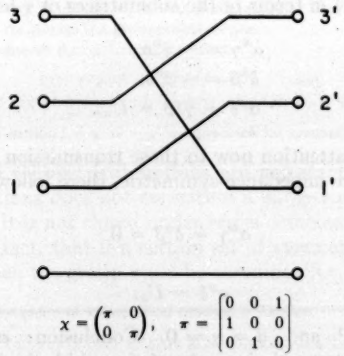
Two additional subgroups of Γ are those with transmission matrices of the form

$$\chi_1 = \begin{pmatrix} \alpha & \beta \\ 0 & (\alpha^*)^{-1} \end{pmatrix}$$

and

$$\chi_2 = \begin{pmatrix} \alpha & 0 \\ \gamma & (\alpha^*)^{-1} \end{pmatrix},$$

respectively. The matrix χ_1 corresponds to the series combination of a transformer bank and a series impedance transmission network, and the resulting group is simply that generated by the transformer bank sub-



$$\chi = \begin{pmatrix} \pi & 0 \\ 0 & \pi \end{pmatrix}, \quad \pi = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

FIG. 9.

group and the series impedance subgroup. A similar relationship exists between χ_2 and the parallel impedance subgroup.

Considerations of instantaneous power absorption by a passive transmission network lead to another general subgroup of Γ. The instantaneous absorbed power P_a (see footnote 3) is by definition

$$P_a = e^* i - (e')^* i'.$$

In terms of the symmetrical matrix

$$H = \begin{pmatrix} 0 & U_n \\ U_n & 0 \end{pmatrix}$$

this can be expressed as the quadratic form

$$P_a = \tfrac{1}{2}\, t^* (H - \chi^* H \chi) t$$

in the input vector t. A necessary and sufficient condition that P_a vanish identically is thus

$$\chi^* H \chi = H, \tag{11}$$

which clearly forces χ to be non-singular. Since this condition is satisfied by χ^{-1} whenever it is satisfied by χ and by $\chi_2\chi_1$ whenever it is satisfied by each factor, the set of all such matrices χ is a group. Moreover, from (11) we find $\det \chi = \pm 1$. We have thereby established that *the set of all passive transmission networks for which the instantaneous absorbed power is identically zero forms an extended unimodular subgroup of* Γ.

Evaluating (11) in terms of the submatrices of χ leads to

$$\begin{aligned} \alpha^*\gamma &= -\gamma^*\alpha, \\ \delta^*\beta &= -\beta^*\delta, \\ \alpha^*\delta + \gamma^*\beta &= U_n. \end{aligned}$$

If we restrict our attention now to those transmission networks of this subgroup which are impedance-symmetric, there follows by application of theorem 3

$$\alpha^*\gamma = \delta^*\beta = 0$$

and

$$\alpha^*\delta = U_n.$$

Hence, $\delta = (\alpha^*)^{-1}$ and $\beta = \gamma = 0$. Conclusion: *every impedance-symmetric passive transmission network for which the instantaneous absorbed power vanishes identically is a transformer bank.*

6. SYMMETRICAL SECTIONS

A type of symmetry of importance in the theory of transmission networks is that characterized by invariance of the section under interchange of corresponding input and output terminals, or, equivalently, under interchange of e with e' and i with $-i'$.

Definition 14. A transmission network

$$t' = \chi t + \tau \tag{12}$$

will be called a *symmetrical section* (or said to have *section symmetry*) provided all terminal vectors t and t' satisfying (12) also satisfy

$$Jt = \chi J t' + \tau, \tag{13}$$

where J is the diagonal matrix

$$J = \begin{pmatrix} U_n & 0 \\ 0 & -U_n \end{pmatrix}.$$

By eliminating i' there results

Theorem 5. A necessary and sufficient condition that a transmission network $i' = \chi i + \tau$ be a symmetrical section is that χ and τ satisfy

$$(1)\quad (J\chi)^2 = U_{2n} \qquad \text{and} \qquad (2)\quad (\chi J + U_{2n})\tau = 0.$$

In terms of the submatrices of χ condition (1) becomes

$$\begin{cases} \alpha^2 = U_n + \beta\gamma, \\ \delta^2 = U_n + \gamma\beta, \\ \alpha\beta = \beta\delta, \\ \gamma\alpha = \delta\gamma. \end{cases} \tag{14}$$

Corollary 5.1. Every symmetrical section is non-singular (having, in fact, determinant ± 1), and its inverse is a symmetrical section.

Proof. We observe that $\chi^{-1} = J\chi J$, so that

$$(1)\quad (J\chi^{-1})^2 = J(J\chi)^2 J = U_{2n} \qquad \text{and}$$
$$(2)\quad (\chi^{-1}J + U_{2n})(-\chi^{-1}\tau) = -\chi^{-1}(\chi J + U_{2n})\tau = 0.$$

Thus, the inverse section $i = \chi^{-1}i' - \chi^{-1}\tau$ must itself be symmetrical.

Even though all symmetrical sections are non-singular, the set of all symmetrical sections does not constitute a subgroup of Γ, since (as is readily verified) it is not closed under series combination.

We show, in fact, that if a certain set of symmetrical sections does form a group, then the group must be commutative.

Theorem 6. Every group of symmetrical sections is Abelian.

Proof. It is convenient here to write a $2n$-dimensional affine transformation of the form $i' = \chi i + \tau$ as the $(2n+1)$-dimensional linear transformation

$$\begin{pmatrix} i' \\ 1 \end{pmatrix} = \begin{pmatrix} \chi & \tau \\ 0 & 1 \end{pmatrix}\begin{pmatrix} i \\ 1 \end{pmatrix},$$

which we shorten to $\bar{i}' = \bar{\chi}\bar{i}$. In this notation the symmetry criterion of theorem 5 becomes simply $\bar{\chi}^{-1} = \bar{J}\bar{\chi}\bar{J}$, where

$$\bar{J} = \begin{pmatrix} J & 0 \\ 0 & 1 \end{pmatrix}.$$

Now, if $\bar{\chi}_1$ and $\bar{\chi}_2$ correspond to elements N_1 and N_2 of a group of symmetrical sections, then by the above criterion we have

$$(\bar{\chi}_2\bar{\chi}_1)^{-1} = \bar{J}(\bar{\chi}_2\bar{\chi}_1)\bar{J}$$

and

$$(\bar{\chi}_2\bar{\chi}_1)^{-1} = \bar{\chi}_1^{-1}\bar{\chi}_2^{-1} = \bar{J}\bar{\chi}_1\bar{J}\bar{\chi}_2\bar{J}.$$

There follows $\bar{\chi}_2\bar{\chi}_1 = \bar{\chi}_1\bar{\chi}_2$, establishing the commutativity of N_1 and N_2.

The case of γ non-singular merits special attention, since (as is proved later) non-singularity of γ is essential for the existence of a characteristic impedance.

Theorem 7. Let

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

be the transmission matrix of a passive symmetrical section N. If γ is non-singular, then

(1) $\det \chi = 1$;

(2) $$\chi = \begin{pmatrix} \alpha & (\alpha^2 - U_n)\gamma^{-1} \\ \gamma & \gamma\alpha\gamma^{-1} \end{pmatrix} = \begin{pmatrix} U_n & (\alpha - U_n)\gamma^{-1} \\ 0 & U_n \end{pmatrix} \times \begin{pmatrix} U_n & 0 \\ \gamma & U_n \end{pmatrix} \begin{pmatrix} U_n & (\alpha - U_n)\gamma^{-1} \\ 0 & U_n \end{pmatrix};$$

(3) N corresponds to the canonical fixed-terminal network having impedance matrix

$$Z = \begin{pmatrix} A & C \\ C & A \end{pmatrix},$$

where $A = \alpha\gamma^{-1}$ and $C = \gamma^{-1}$.

Proof. By eliminating β and δ in Eqs. 14 we obtain the first equality in (2), and the indicated factorization of χ is immediate. Assertion (1) is obvious from this factorization, or, alternatively, from lemma 1. Finally, (3) follows directly from definition 11.

Observe that the factorization in (2) can be interpreted as yielding a representation of N as a generalized symmetrical **T**-section, since the middle factor corresponds to a parallel impedance transmission network, and each of the equal outer factors to a series impedance transmission network.

To be concluded.

ASYMPTOTIC EXPANSION OF THE WHITTAKER'S FUNCTION $W_{k,m}(z)$ FOR LARGE VALUES OF k, m, z

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Part II[3]

APPENDIX A—REDUCTION OF THE COMPLEX ASYMPTOTIC FORM OF $W^{(n)}_{k,m}(z)$ TO THE REAL FORM

In order to reduce Eq. 25 to a real form and to maintain a consistent order of magnitude, we shall expand the various quantities involved in the equation in descending power series of n. Thus (referring to Eq. 2)

$$k = -nk_0 + k_1 \tag{A-1a}$$

$$m = inm_0 + m_1 + \frac{i}{8m_0}\left(\frac{1}{n}\right) + O\left(\frac{1}{n^3}\right) \quad (n > 1) \tag{A-1b}$$

$$z = nz_0 \tag{A-1c}$$

where

$$k_0 = \frac{c - 2b}{2c^2}\sqrt{\frac{a}{b}} \tag{A-1d}$$

$$m_0 = \frac{\sqrt{a(c-b)}}{c^2} \tag{A-1e}$$

$$z_0 = \frac{2\sqrt{ab}}{c^2}(1 + c\sigma) \tag{A-1f}$$

are all independent of n. k_1, m_1, as mentioned already, are two arbitrary constants introduced to make the analysis more general. Substituting Eq. A-1 in the expression for Δ^2, namely

$$\Delta^2 = \left(1 - \frac{4k}{z} + \frac{4m^2}{z^2}\right) - \frac{4m - 1}{z^2}$$

we obtain

$$\Delta^2 = \Delta_0^2 + \frac{4im_0(2m_1 - 1) - 4k_1z_0}{z_0^2}\left(\frac{1}{n}\right) + \frac{(2m_1 - 1)^2 + 1}{z_0^2}\left(\frac{1}{n^2}\right) + O\left(\frac{1}{n^3}\right) \tag{A-2}$$

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[3] Part I appeared in this JOURNAL for March, 1953, p. 215.

where

$$\Delta_0^2 = 1 + \frac{4k_0}{z_0} - \frac{4m_0^2}{z_0^2} = \frac{c^2}{2b} K(\sigma) = k(\sigma) \tag{A-3}$$

$$K(\sigma) = \frac{a\sigma(1 + b\sigma)}{(1 + c\sigma)^2}. \tag{A-4}$$

Note that $\Delta_0 = 0$ when $\sigma = 0$, that is, at the sonic flow, while for subsonic flow $\Delta_0^2 > 0$, supersonic flow $\Delta_0^2 < 0$. Hence for non-sonic flow $\Delta_0 \neq 0$, so that

$$\Delta = \Delta_0 + \frac{2im_0(2m_1 - 1) - 2k_1z_0}{\Delta_0 z_0^2}\left(\frac{1}{n}\right) + \frac{\Delta_0^2 z_0^2[(2m_1 - 1)^2 + 1] - 4[im_0(2m_1 - 1) - k_1z_0]^2}{2\Delta_0^3 z_0^4} \times \left(\frac{1}{n^2}\right) + O\left(\frac{1}{n^3}\right) \tag{A-5}$$

and the saddle points are

$$t_{1,2} = \lambda_{1,2}^{(0)} + \frac{\lambda_{1,2}^{(1)}}{n} + \frac{\lambda_{1,2}^{(2)}}{n^2} + O\left(\frac{1}{n^3}\right), \tag{A-6a}$$

where

$$\lambda_{1,2}^{(0)} = \frac{im_0}{z_0} - \frac{1}{2} \pm \frac{\Delta_0}{2} \tag{A-6b}$$

$$\lambda_{1,2}^{(1)} = \frac{(m_1 - \frac{1}{2})\Delta_0 z_0 \pm [im_0(2m_1 - 1) - k_1z_0]}{\Delta_0 z_0^2} \tag{A-6c}$$

$$\lambda_{1,2}^{(2)} = \frac{-i}{8m_0z_0} \pm \frac{\Delta_0^2 z_0^2[(2m_1 - 1) + 1] - 4[im_0(2m_1 - 1) - k_1z_0]^2}{4\Delta_0^3 z_0^4}. \tag{A-6d}$$

For sonic flow $\Delta_0 = 0$ and we have

$$\Delta = \frac{2}{z_0}\sqrt{\frac{im_0(2m_1 - 1) - k_1z_0}{n}} \times \left[1 + \frac{1}{8}\left\{\frac{(2m_1 - 1)^2 + 1}{im_0(2m_1 - 1) - k_1z_0}\right\}\left(\frac{1}{n}\right) + O\left(\frac{1}{n^2}\right)\right] \tag{A-7}$$

and the saddle points are given by

$$t_{1,2} = \mu^{(0)} + \frac{\mu_{1,2}^{(\frac{1}{2})}}{n^{\frac{1}{2}}} + \mu^{(1)}\left(\frac{1}{n}\right) + \mu_{1,2}^{(\frac{1}{2})}\left(\frac{1}{n^{\frac{1}{2}}}\right) + O\left(\frac{1}{n^2}\right), \tag{A-8a}$$

where

$$\mu^{(0)} = -\frac{1}{2} + \frac{im_0}{z_0} \tag{A-8b}$$

$$\mu_{1,2}^{(\frac{1}{2})} = \pm\sqrt{i\frac{m_0}{z_0}\frac{(2m_1-1)}{z_0} - \frac{k_1}{z_0}} \tag{A-8c}$$

$$\mu^{(1)} = \frac{m_1 - \frac{1}{2}}{z_0} \tag{A-8d}$$

$$\mu_{1,2}^{(\frac{3}{2})} = \pm\frac{1}{z_0}\frac{(2m_1-1)^2+1}{\sqrt{im_0(2m_1-1) - k_1 z_0}}. \tag{A-8e}$$

As $n \to \infty$,

$$t_{1,2} = \lambda_{1,2}^{(0)} \tag{A-9}$$

for $\Delta_0 \neq 0$ while for $\Delta_0 = 0$, we have $t_1 = t_2 = \mu^{(0)}$. Hence, the two saddle points coincide at $\mu^{(0)} = \dfrac{im_0}{z_0} - \dfrac{1}{2}$ for sonic flow. It can be readily verified that

$$\frac{k_0 + im_0}{\lambda_{1,2}^{(0)}} + \frac{-k_0 + im_0}{1 + \lambda_{1,2}^{(0)}} - z_0 = 0 \tag{A-10}$$

and

$$\frac{k_0 + im_0}{\mu^{(0)}} + \frac{-k_0 + im_0}{1 + \mu^{(0)}} - z_0 = 0. \tag{A-11}$$

Substituting $t_{1,2}$ in $F(t_{1,2})$ and remembering the above identities, we obtain, for $\Delta_0 \neq 0$ (subsonic and supersonic speeds)

$$F(t_{1,2}) = (-\lambda_{1,2}^{(0)})^{-k-\frac{1}{2}+m}(1+\lambda_{1,2}^{(0)})^{k-\frac{1}{2}+m}e^{-z\lambda_{1,2}^{(0)}}\left\{1 + O\left(\frac{1}{n}\right)\right\} \tag{A-12}$$

and for $\Delta_0 = 0$ (sonic speed)

$$F(t_{1,2}) = (-\mu^{(0)})^{-k-\frac{1}{2}+m}(1+\mu^{(0)})^{k-\frac{1}{2}+m}e^{-z\mu^{(0)}}\left\{1 + O\left(\frac{1}{n}\right)\right\}. \tag{A-13}$$

In the following we shall consider these cases separately.

I. $\Delta_0 \neq 0$, *Subsonic and Supersonic Flow*

Substituting the value of $\lambda_{1,2}^{(0)}$ in Eq. A-12, we obtain

$$F(t_{1,2}) = \left[-\left(\frac{k_0}{z_0} - \frac{2m_0^2}{z_0^2} \pm \frac{im_0\Delta_0}{z_0}\right)\right]^{m-\frac{1}{2}}$$

$$\times\left\{\frac{\frac{1}{2} \pm \frac{\Delta_0}{2} + \frac{im_0}{z_0}}{-\left(-\frac{1}{2} \pm \frac{\Delta_0}{2} + \frac{im_0}{z_0}\right)}\right\}^k e^{\frac{1\mp\Delta_0}{2}z\pm i\pi m_0}\left[1 + O\left(\frac{1}{n}\right)\right]. \tag{A-14}$$

Using Eq. A-6, we obtain also

$$\sqrt{\frac{t_1(t_1+1)}{t_1-t_2}} = \sqrt{\frac{\lambda_1^{(0)}(1+\lambda_1^{(0)})}{\lambda_1^{(0)}-\lambda_2^{(0)}}}\left[1+O\left(\frac{1}{n}\right)\right]$$

$$= \sqrt{\frac{1}{\Delta_0}\left(\frac{k_0}{z_0}-\frac{2m_0^2}{z_0^2}+\frac{im_0\Delta_0}{z_0}\right)}\left[1+O\left(\frac{1}{n}\right)\right] \quad \text{(A-15}a\text{)}$$

$$\sqrt{\frac{t_2(t_2+1)}{t_2-t_1}} = \sqrt{\frac{\lambda_2^{(0)}(1+\lambda_2^{(0)})}{\lambda_2^{(0)}-\lambda_1^{(0)}}}\left[1+O\left(\frac{1}{n}\right)\right]$$

$$= \sqrt{-\frac{1}{\Delta_0}\left(\frac{k_0}{z_0}-\frac{2m_0^2}{z_0^2}+\frac{im_0\Delta_0}{z_0}\right)}\left[1+O\left(\frac{1}{n}\right)\right]. \quad \text{(A-15}b\text{)}$$

Finally, it can be verified that

$$z^m = z^{inm_0+m_1}\left[1+O\left(\frac{\log n}{n}\right)\right]$$

$$(k-\tfrac{1}{2}-m)^{k-m} = [-n(k_0+im_0)]^{k-inm_0-m_1}e^{k_1-m_1-\frac{1}{2}}\left[1+O\left(\frac{\log n}{n}\right)\right]$$

$$e^{-\left(k+\frac{s-1}{2}-m\right)} = e^{-k+inm_0+m_1-\frac{s}{2}}\left[1+O\left(\frac{\log n}{n}\right)\right]$$

so that

$$z^m(k-\tfrac{1}{2}-m)^{k-m}e^{-\left(k+\frac{s-1}{2}-m\right)} = z^k\left[-\left(\frac{k_0}{z_0}+i\frac{m_0}{z_0}\right)\right]^{k-inm_0-m_1}$$

$$\times e^{nk_0+inm_0-\frac{ns\Delta_0}{2}}\left[1+O\left(\frac{\log n}{n}\right)\right]. \quad \text{(A-16)}$$

Substituting Eqs. A-14, A-15, A-16 in Eq. 25, and remembering that Δ_0 is real for subsonic flow and its square is equal to $1+\frac{4k_0}{z_0}-\frac{4m_0^2}{z_0^2}$, one obtains the asymptotic expansion of $W_{k,m}^{(n)}(z)$ *for subsonic flow*

$$W_{k,m}^{(n)}(z) \sim \frac{-z^k e^{nk_0-\frac{s\Delta_0}{2}}}{\sqrt{\Delta_0}}\left\{\frac{\left(\frac{1}{2}+\frac{k_0}{z_0}-\frac{\Delta_0}{2}\right)}{\left(-\frac{1}{2}+\frac{k_0}{z_0}-\frac{\Delta_0}{2}\right)}\right\}^{\frac{k}{2}}\left[\frac{k_0^2}{z_0^2}+\frac{m_0^2}{z_0^2}\right]^{\frac{k}{2}}$$

$$\times \exp\left\{-nm_0\tan^{-1}\left[\frac{\frac{m_0k_0}{z_0^2}(\Delta_0-1)+\frac{2m_0^3}{z_0^3}}{\frac{k_0^2+m_0^2\Delta_0}{z_0^2}-2\frac{k_0m_0^2}{z_0^2}}\right]\right\}$$

$$\times\left[1+O\left(\frac{\log n}{n}\right)\right], \quad \text{(A-17)}$$

where m_1 is taken to be zero. The above expression is real and is useful in numerical calculation.

On the other hand, for supersonic flow, $\Delta_0^2 < 0$ so that Δ_0 is imaginary. Define

$$\Delta_0 = i\Delta_1. \tag{A-18}$$

Substituting Eqs. A-14, A-15, A-16, A-18 in Eq. 32, we have

$$W_{k,m}^{(n)}(z) \sim -i\frac{z^k e^{nk_0 - i\frac{z_0\Delta_1}{2}n}}{\sqrt{-i\Delta_1}}\left\{\frac{\frac{k_0}{z_0} - \frac{2m_0^2}{z_0^2} - \frac{m_0\Delta_1}{z_0}}{\frac{k_0}{z_0} + i\frac{m_0}{z_0}}\right\}^{im_0+m_1}$$

$$\times\left[\frac{\frac{1}{2} + \frac{im_0}{z_0} + \frac{i\Delta_1}{2}}{-\frac{1}{2} + \frac{im_0}{z_0} + \frac{i\Delta_1}{2}}\left(\frac{k_0}{z_0} + \frac{im_0}{z_0}\right)\right]^k + i\frac{z^k e^{nk_0 + i\frac{z_0\Delta_1}{2}n}}{\sqrt{i\Delta_1}}$$

$$\times\left\{\frac{\frac{k_0}{z_0} - \frac{2m_0^2}{z_0^2} + \frac{m_0\Delta_1}{z_0}}{\frac{k_0}{z_0} + i\frac{m_0}{z_0}}\right\}^{im_0+m_1}$$

$$\times\left[\frac{\frac{1}{2} + \frac{im_0}{z_0} - \frac{i\Delta_1}{2}}{-\frac{1}{2} + \frac{im_0}{z_0} - \frac{i\Delta_1}{2}}\left(\frac{k_0}{z_0} + \frac{im_0}{z_0}\right)\right]^k. \tag{A-19}$$

Remembering that Δ_1 is now real and its square is given by $\frac{4m_0^2}{z_0^2} - \frac{4k_0}{z_0} - 1$, it can be readily verified that

$$\frac{\frac{k_0}{z_0} - \frac{2m_0^2}{z_0^2} + \frac{m_0\Delta_1}{z_0}}{\frac{k_0}{z_0} + i\frac{m_0}{z_0}} = \left[\frac{\frac{k_0}{z_0} - \frac{2m_0^2}{z_0^2} - \frac{m_0\Delta_1}{z_0}}{\frac{k_0}{z_0} - i\frac{m_0}{z_0}}\right]^{-1} \tag{A-20a}$$

$$\frac{\frac{1}{2} + i\frac{m_0}{z_0} - i\frac{\Delta_1}{2}}{-\frac{1}{2} + i\frac{m_0}{z_0} - i\frac{\Delta_1}{2}}\left(\frac{k_0}{z_0} + i\frac{m_0}{z_0}\right)$$

$$= \frac{\frac{1}{2} - i\frac{m_0}{z_0} - i\frac{\Delta_1}{2}}{-\frac{1}{2} - i\frac{m_0}{z_0} - i\frac{\Delta_1}{2}}\left(\frac{k_0}{z_0} - i\frac{m_0}{z_0}\right) \tag{A-20b}$$

so that the two terms of Eq. A-19 are complex conjugates. We thus obtain for *the supersonic case* a real expression for the asymptotic expansion of $W^{(\varkappa)}_{k,m}(z)$

$$W^{(\varkappa)}_{k,m}(z) \sim -\frac{2z^k}{\sqrt{\Delta_1}}\left[\frac{k_0^2}{z_0^2}+\frac{m_0^2}{z_0^2}\right]^{\frac{k}{2}} e^{nk_0-nm_0\tan^{-1}\left(\frac{m_0}{-k_0}\right)}$$

$$\times\cos\left\{nm_0\log\left[\frac{-\frac{k_0}{z_0}+\frac{2m_0^2}{z_0^2}+\frac{m_0\Delta_1}{z_0}}{\frac{k_0^2}{z_0^2}+\frac{m_0^2}{z_0^2}}\right]-\frac{z\Delta_1}{2}-(k+\tfrac{1}{4})\pi\right.$$

$$\left.+2k\tan^{-1}2\left(\frac{m_0}{z_0}+\frac{\Delta_1}{2}\right)-k\tan^{-1}\frac{m_0}{z_0}\right\}. \quad \text{(A-21)}$$

It should be noted that $\tan^{-1}\left(\frac{m_0}{-k_0}\right)$ indicates the angle is in the second quadrant.

II. $\Delta_0 = 0$, *Sonic Flow*

We have already shown that for $\Delta_0 = 0$

$$t_{1,2} = \mu^{(0)} = \left(-\frac{1}{2}+\frac{im_0}{z_0}\right)\left[1+O\left(\frac{1}{\sqrt{n}}\right)\right] \quad \text{(A-8)}$$

$$F(t_{1,2}) = (-\mu^{(0)})^{-k-\frac{1}{2}+m}(1+\mu^{(0)})^{k-\frac{1}{2}+m}e^{-z\mu^{(0)}}\left[1+O\left(\frac{1}{\sqrt{n}}\right)\right]. \quad \text{(A-13)}$$

Now

$$z^m(k-\tfrac{1}{2}-m)^{k-m}e^{-\left(k+\frac{z-1}{2}-m\right)}$$

$$= z^k\left[-\frac{k_0}{z_0}-i\frac{m_0}{z_0}\right]^{k-im_0-m}e^{nk_0+inm_0-\frac{z}{2}}\left[1+O\left(\frac{\log n}{n}\right)\right] \quad \text{(A-16)}$$

and also

$$\sqrt[3]{|t_1(t_1+1)|} = \sqrt[3]{\frac{1}{4}+\frac{m_0^2}{z_0^2}}. \quad \text{(A-22)}$$

Substituting these expressions in Eq. 42*b*, we obtain

$$W^{(\varkappa)}_{k,m}(z) \sim \frac{n^{k+\frac{1}{3}}}{\sqrt{2\pi}}\Gamma(\tfrac{1}{3})\left(\frac{z_0}{3}\right)^{\frac{1}{3}}e^{nk_0-2nm_0\tan^{-1}\frac{2m_0}{z_0}}\left(\frac{m_0^2}{z_0^2}+\frac{1}{4}\right)^{k-\frac{1}{3}}. \quad \text{(A-23)}$$

APPENDIX B—ASYMPTOTIC EXPANSION OF $W_{k,m}^{(n)}(z)$ FOR LARGE VALUE OF n OBTAINED BY APPLYING LANGER'S THEORY

According to Langer's theory (5)[4], a neat summary of which was recently given by Lighthill (6), the two asymptotic solutions of the differential equation

$$\frac{d^2u}{dy^2} + p(y)\frac{du}{dy} + [n^2q(y) + r(y)]u = 0 \tag{B-1}$$

where $p(y)$, $q(y)$ and $r(y)$ are twice differentiable, which are valid, as $|n| \to \infty$ for all arg. of n are

$$u_1(y) = \exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right)|s|^{\frac{1}{4}}|q|^{-\frac{1}{4}}(ns)^{\frac{1}{4}}J_{-\frac{1}{4}}(ns)\sqrt{2\pi} \tag{B-2a}$$

$$u_2(y) = \exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right) \times |s|^{\frac{1}{4}}|q|^{-\frac{1}{4}}sgn(y - y_1)(ns)^{-\frac{1}{4}}J_{\frac{1}{4}}(ns)\sqrt{2\pi}, \tag{B-2b}$$

where

$$s = \int_{y_1}^{y}\sqrt{q}\,dy, \quad q(y_1) = 0. \tag{B-3}$$

When $|ns|$ is large, Eq. (B-2) reduces to

$$u_1(y) \sim \exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right)\frac{|s|^{\frac{1}{4}}}{|q|^{\frac{1}{4}}}\left[\frac{e^{ins}}{(ins)^{\frac{1}{4}}} + \frac{e^{-ins}}{(-ins)^{\frac{1}{4}}}\right]$$

$$u_2(y) \sim \exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right)\frac{|s|^{\frac{1}{4}}sgn(y - y_1)}{|q|^{\frac{1}{4}}}\left[\frac{e^{ins}}{(ins)^{\frac{1}{4}}} + \frac{e^{-ins}}{(-ins)^{\frac{1}{4}}}\right]$$

so that for large value of $|ns|$, the general solution is some combination of solutions of the forms:

$$\exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right)|q|^{-\frac{1}{4}}e^{ins} \tag{B-4a}$$

$$\exp.\left(-\frac{1}{2}\int_{y_1}^{y} p\,dy\right)|q|^{-\frac{1}{4}}e^{-ins}. \tag{B-4b}$$

The differential equation considered in the present paper is of the form

$$\frac{d^2F}{d\sigma^2} - n^2K(\sigma)F = 0, \tag{B-5}$$

where

$$K(\sigma) = \frac{a\sigma(1 + b\sigma)}{(1 + c\sigma)^2}, \tag{B-6}$$

[4] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

$K(\sigma)$ being positive when $\sigma > 0$ (subsonic flow) and negative when $\sigma < 0$ (supersonic). Hence, in our case

$$\begin{cases} y = \sigma, \quad y_1 = 0 \\ p(\sigma) = 0 \\ q(\sigma) = -K(\sigma) \\ r(\sigma) = 0 \\ \exp.\left(-\frac{1}{2}\int_0^\sigma 0 d\sigma\right) = 1 \\ s(\sigma) = \int_0^\sigma \sqrt{-K(\sigma)}\, d\sigma. \end{cases} \tag{B-7}$$

Hence for large value of $|ns|$—that is, for high supersonic and low subsonic speeds for which s is not small—the asymptotic solution of F in Eq. B-5 will be some combination of the forms

$$|K(\sigma)|^{-\frac{1}{4}} e^{in\int_0^\sigma \sqrt{-K(\sigma)}d\sigma}$$

$$|K(\sigma)|^{-\frac{1}{4}} e^{-in\int_0^\sigma \sqrt{-K(\sigma)}d\sigma},$$

that is,

$$F(n, \sigma) \sim |K(\sigma)|^{-\frac{1}{4}}\left[f_1(n)e^{in\int_0^\sigma \sqrt{-K(\sigma)}d\sigma} + f_2(n)e^{-in\int_0^\sigma \sqrt{-K(\sigma)}d\sigma}\right]. \tag{B-8}$$

We shall determine that solution of Eq. B-5 which remains finite as $\sigma \to \infty$. It is known (1) that a, b, c are chosen such that as $\sigma \to \infty$, $K(\sigma) \to 1$. Consequently as n increases indefinitely the asymptotic solution of that solution of Eq. B-5 which remains finite as $\sigma \to \infty$ will be

$$F(n, \sigma) \sim |K(\sigma)|^{-\frac{1}{4}} f_1(n) e^{in\int_0^\sigma \sqrt{-K(\sigma)}d\sigma}. \tag{B-9}$$

This asymptotic formula is valid for all σ.

Now it has been shown (1) that Eq. B-5 is reducible to Whittaker's equation for which a solution that remains finite as $\sigma \to \infty$ is known to be $W^{(n)}_{k,m}(z)$ where

$$\begin{aligned} k &= -\frac{c - 2b}{2c^2}\sqrt{\frac{a}{b}}\, n \\ m &= \sqrt{\frac{1}{4} - \frac{n^2 a(c-b)}{c^4}} \\ z &= 2n\frac{\sqrt{ab}}{c^2}(1 + c\sigma). \end{aligned} \tag{B-10}$$

We need now only to identify the function $F(n, y)$ with the Whittaker

function $W^{(n)}_{k,m}(z)$. We note that for large σ but finite n, the leading term in the asymptotic expansion of the Whittaker function is (2)

$$W^{(n)}_{k,m}(z) \sim e^{-\frac{1}{2}z}z^k = [2n\sigma]^{-\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}}n}e^{-\frac{n}{c}}e^{-n\sigma} \tag{B-11}$$

as $\frac{\sqrt{ab}}{c^2} = 1$. This formula is valid for any n, *however large*, provided that $\sigma \gg n^2$.

On the other hand, one should be able to obtain a similar expression by letting $\sigma \to \infty$ in Eq. B-9. Now

$$\int_0^\sigma \sqrt{\frac{\sigma(1+b\sigma)}{(1+c\sigma)^2}}\,d\sigma = \frac{c-2b}{2c^2\sqrt{b}}\log\left(1+2b\sigma+\sqrt{(1+2b\sigma)^2-1}\right) + \frac{\sqrt{\sigma(1+b\sigma)}}{c} - \frac{\sqrt{c-b}}{c^2}\sin^{-1}\frac{\sigma(c-2b)-1}{1+c\sigma} - \frac{\pi}{2}\frac{\sqrt{c-b}}{c^2}$$

$$e^{in\int_0^\sigma\sqrt{-K(\sigma)}\,d\sigma} = [4b\sigma]^{-n\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}}}e^{-n\sigma-\frac{n}{2b}+\frac{n}{c}\sqrt{\frac{c-b}{b}}\sin^{-1}\frac{c-2b}{c}+\frac{n}{c}\sqrt{\frac{c-b}{b}}\frac{\pi}{2}}\left[1+O\left(\frac{1}{\sigma}\right)\right]$$

$$K(\sigma) = 1 + O\left(\frac{1}{\sigma}\right).$$

Hence Eq. B-9 becomes

$$F(n,\sigma) \sim f_1(n)e^{-\frac{n}{2b}+\frac{n}{c}\sqrt{\frac{c-b}{b}}\left(\sin^{-1}\frac{c-2b}{c}+\frac{\pi}{2}\right)} \times [4b\sigma]^{-n\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}}}e^{-n\sigma}\left[1+O\left(\frac{1}{\sigma}\right)\right] \tag{B-12}$$

as $n_1\sigma \to \infty$. Comparing Eq. B-12 with Eq. B-11, we conclude

$$f_1(n) = \left[\frac{2b}{n}\right]^{n\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}}}e^{n\frac{c-2b}{2bc}-\frac{n}{c}\sqrt{\frac{c-b}{b}}\left(\sin^{-1}\frac{c-2b}{c}+\frac{\pi}{2}\right)}. \tag{B-13}$$

Consequently, the solution of Eq. B-5 which reduces to the Whittaker function $W^{(n)}_{k,m}(z)$ has the following asymptotic form for $n \to \infty$ and $\sigma \gg 0$:

$$F(n,\sigma) \sim \left[\frac{2be}{n}\right]^{n\frac{c-2b}{2bc}}e^{-\frac{n}{c}\sqrt{\frac{c-b}{b}}\left(\sin^{-1}\frac{c-2b}{c}+\frac{\pi}{2}\right)}|K(\sigma)|^{-\frac{1}{4}}e^{-n\int_0^\sigma\sqrt{k(\sigma)}\,d\sigma} \tag{B-14}$$

where

$$|K(\sigma)|^{-\frac{1}{4}} = \left|\frac{a\sigma(1+b\sigma)}{(1+c\sigma)^2}\right|^{-\frac{1}{4}} > 0, \quad \left|\sin\frac{c-2b}{c}\right| \le \frac{\pi}{2}$$

and

$$\int_0^\sigma \sqrt{K(\sigma)}\, d\sigma = \frac{c-2b}{2bc}\log\left(1+2b\sigma+\sqrt{(1+2b\sigma)^2-1}\right) + \frac{\sqrt{a\sigma(1+b\sigma)}}{c} - \frac{1}{c}\sqrt{\frac{c-b}{b}}\left(\sin^{-1}\frac{\sigma(c-2b)-1}{1+c\sigma}+\frac{\pi}{2}\right). \quad \text{(B-15)}$$

Since both Eq. 26 and Eq. B-13 represent the asymptotic expansion of the Whittaker function $W_{k,m}^{(n)}(z)$ as $n \to \infty$ for values of $\sigma \gg 0$, they should be reducible to each other. In fact this has been found to be so. It is not difficult to show that Eq. 26 can be simplified first into the form

$$W_{k,m}^{(n)}(z) \sim \frac{n^k}{\sqrt{\Delta_0}}\left[\frac{1+\Delta_0}{2}z_0+k_0\right]^k e^{nk_0-n\frac{z_0\Delta_0}{2}+nm_0}\left[\sin^{-1}\frac{k_0-\frac{2m_0^2}{z_0}}{\sqrt{m_0^2+k_0^2}} - \sin^{-1}\frac{k_0}{\sqrt{m_0^2+k_0^2}}\right] \times \left[1+O\left(\frac{\log n}{n}\right)\right] \quad \text{(B-16)}$$

where

$$k = -nk_0 + k_1$$

$$\Delta_0 = \sqrt{1+\frac{4k_0}{z_0}-\frac{4m_0^2}{z_0^2}}$$

and the angles in the formula are chosen according to

$$\left|\sin^{-1}\frac{k_0-\frac{2m_0^2}{z_0}}{\sqrt{m_0^2+k_0^2}}\right|, \quad \left|\sin^{-1}\frac{k_0}{\sqrt{m_0^2+k_0^2}}\right| \leq \frac{\pi}{2}.$$

Remembering that

$$\begin{aligned} c &= \sqrt{ab} \\ k_0 &= \frac{c-2b}{2bc} \\ m_0 &= \frac{1}{c}\sqrt{\frac{c-b}{b}} \\ \Delta_0 &= \sqrt{1+\frac{4k_0}{z_0}-\frac{4m_0^2}{z_0^2}} \\ z_0 &= \frac{2}{c}(1+c\sigma) \end{aligned} \quad \text{(B-17)}$$

it is simple to verify that

$$\Delta_0 = \sqrt{K(\sigma)} = \sqrt{\frac{a\sigma(1 + b\sigma)}{(1 + c\sigma)^2}} \tag{B-18}$$

and that Eq. B-16 is reducible to Eq. B-14 when $k_1 = 0$.

Equation B-14 gives the asymptotic expansion of $W^{(n)}_{k,m}(z)$ as $n \to \infty$ for low subsonic speed (that is, $\sigma \gg 0$). Our next problem is to find the continuation of the asymptotic expansion (B-14) for other ranges of speed, for example, sonic speed ($\sigma = 0$), high supersonic speed ($\sigma \ll 0$), transonic speeds ($\sigma \doteq 0$ which may be high subsonic or low supersonic speed corresponding respectively to $\sigma > 0$ and $\sigma < 0$). For this purpose, we recall that the asymptotic solution of the Eq. B-1 will, in general, be a certain combination of $u_1(y)$ and $u_2(y)$ defined by Eqs. B-2 and B-3. For our case (that is, for the differential equation B-5)

$$u_1(\sigma) = |K(\sigma)|^{-\frac{1}{4}}|s|^{\frac{1}{2}}(ns)^{\frac{1}{2}}J_{-\frac{1}{4}}(ns)\sqrt{2\pi} \tag{B-19a}$$

$$u_2(\sigma) = |K(\sigma)|^{-\frac{1}{4}}|s|^{\frac{1}{2}}(ns)^{-\frac{1}{4}}J_{\frac{1}{4}}(ns)\sqrt{2\pi}\,sgn(\sigma) \tag{B-19b}$$

where

$$s = \int_0^\sigma \sqrt{-K(\sigma)}\,d\sigma.$$

That is,

$$F(n, \sigma) = W^{(n)}_{k,m}(z) = g_1(n)u_1(\sigma) + g_2(n)u_2(\sigma). \tag{B-20}$$

For $\sigma > 0$, s is imaginary $= i|s|$ (as $K(\sigma) > 0$). Moreover, when $\sigma \gg 0$, $|ns|$ will be very large as $n \to \infty$ so that Eq. B-19 becomes

$$u_1(\sigma) \sim |K(\sigma)|^{-\frac{1}{4}}\left[\frac{e^{n|s|}}{(n)^{\frac{1}{2}}} + \frac{e^{-n|s|}}{(-n)^{\frac{1}{2}}}\right] \tag{B-21a}$$

$$u_2(\sigma) \sim |K(\sigma)|^{-\frac{1}{4}}\left[\frac{e^{n|s|}}{(n)^{\frac{1}{2}}} + \frac{e^{-n|s|}}{(-n)^{\frac{1}{2}}}\right] \tag{B-21b}$$

and Eq. B-20 becomes

$$F(n, \sigma) = W^{(n)}_{k,m}(z) \sim |K(\sigma)|^{-\frac{1}{4}}e^{n\int_0^\sigma \sqrt{k(\sigma)}d\sigma}\left[\frac{g_1(n)}{(n)^{\frac{1}{2}}} + \frac{g_2(n)}{(n)^{\frac{1}{2}}}\right] + |K(\sigma)|^{-\frac{1}{4}}e^{-n\int_0^\sigma \sqrt{k(\sigma)}d\sigma}\left[\frac{g_1(n)}{(-n)^{\frac{1}{2}}} + \frac{g_2(n)}{(-n)^{\frac{1}{2}}}\right]. \tag{B-22}$$

Comparing Eq. B-22 with B-9, we conclude

$$\begin{cases} \dfrac{g_1(n)}{(n)^{\frac{1}{2}}} + \dfrac{g_2(n)}{(n)^{\frac{1}{2}}} = 0 & \text{(B-23a)} \\[2ex] \dfrac{g_1(n)}{(-n)^{\frac{1}{2}}} + \dfrac{g_2(n)}{(-n)^{\frac{1}{2}}} = f_1(n) & \text{(B-23b)} \end{cases}$$

where $f_1(n)$ is given by Eq. B-13. Solving for $g_1(n)$ and $g_2(n)$ one finds

$$g_1(n) = 3^{-\frac{1}{2}}n^{\frac{1}{2}}f_1(n) \tag{B-24a}$$

$$g_2(n) = -3^{-\frac{1}{2}}n^{\frac{1}{2}}f_1(n). \tag{B-24b}$$

Substituting Eqs. B-24, B-19 in Eq. B-22 we obtain the general expression of the asymptotic formula of the Whittaker function $W^{(n)}_{k,m}(z)$ which is valid *for all values of* σ

$$W^{(n)}_{k,m}(z) \sim \sqrt{\frac{2\pi}{3}}f_1(n)|K(\sigma)|^{-\frac{1}{4}}\left[|ns|^{\frac{1}{2}}(ns)^{\frac{1}{2}}J_{-\frac{1}{3}}(ns) - sgn\ \sigma|ns|^{\frac{1}{2}}(ns)^{-\frac{1}{2}}J_{\frac{1}{3}}(ns)\right], \tag{B-25}$$

where

$$f_1(n) = \left[\frac{2be}{n}\right]^{n\frac{c-2b}{2bc}} e^{-\frac{n}{c}\sqrt{\frac{c}{b}}\left[\sin^{-1}\frac{c-2b}{c}+\frac{\pi}{2}\right]}$$

and

$$K(\sigma) = \frac{a\sigma(1+b\sigma)}{(1+c\sigma)^2}, \quad s = \int_0^\sigma \sqrt{-K(\sigma)}\,d\sigma.$$

We shall now apply the general asymptotic formula Eq. B-25 to the various special cases. For completeness we shall include the case of low subsonic flow from which the formula Eq. B-25 was actually derived.

1. Low Subsonic Flow, $\sigma \gg 0$

$$F(n,\sigma) \sim f_1(n)e^{in\int_0^\sigma\sqrt{-K(\sigma)}d\sigma}|K(\sigma)|^{-\frac{1}{4}}. \tag{B-26}$$

2. High Supersonic Flow, $-\frac{1}{c} < \sigma \ll 0$

When $-\frac{1}{c} < \sigma \ll 0$, s will not be small so that $|ns|$ will be large when $n \to \infty$. Remembering that $z^{-\nu}J_\nu(z)\sqrt{2\pi} \sim \frac{e^{iz}}{(iz)^{\nu+\frac{1}{2}}} + \frac{e^{-iz}}{(-iz)^{\nu+\frac{1}{2}}}$ as $z \to \infty$ and that when $\sigma < 0$, $s\left[= \int_0^\sigma \sqrt{-K(\sigma)}\,d\sigma\right]$ becomes real and positive (that is, $s = +|s|$), Eq. B-25 can be readily reduced to

$$W^{(n)}_{k,m}(z) \sim 2f_1(n)|K(\sigma)|^{-\frac{1}{4}}\cos\left[n\int_0^\sigma\sqrt{-K(\sigma)}\,d\sigma + \frac{\pi}{4}\right]. \tag{B-27}$$

Again it can be verified that Eq. 33 can be reduced to Eq. B-27 if $k_1 = 0$.

3. Sonic Flow ($\sigma = 0$)

When $\sigma \to 0$, then $s \to 0$ and Eq. 25 can be readily reduced to

$$W^{(n)}_{k,m}(z) \sim \frac{1}{\sqrt{\pi}} \Gamma(\tfrac{1}{3}) \left| \frac{n}{3a} \right|^{\frac{1}{3}} f_1(n). \tag{B-28}$$

Again Eq. 42 can be reduced to Eq. B-28 when m_0, k_0, z_0 are expressed in terms of a, b, c and k_1 is put equal to zero.

4. Transonic Range—High Subsonic Speed ($\sigma \doteq 0$ and is Positive)

Here s is imaginary and small. Writing $s = i|s|$, and remembering that

$$I_{-\nu}(z) - I_{\nu}(z) = \frac{2}{\pi} \sin \nu\pi K_{\nu}(z),$$

when I_{ν}, K_{ν} are modified Bessel functions of first and second kind of order ν, Eq. 25 can be simplified to

$$W^{(n)}_{k,m}(z) \sim \sqrt{\frac{2}{\pi}} f_1(n) [K(\sigma)]^{-\frac{1}{4}} K_{\frac{1}{3}}(n|s|). \tag{B-29}$$

5. Transonic Range—Low Supersonic Speed ($\sigma \doteq 0$ and is Negative)

Noting that s is now a real negative number and remembering the formulas

$$Y_{\nu}(z) = \frac{J_{\nu}(z) \cos \nu\pi - J_{-\nu}(z)}{\sin \nu\pi}$$

$$H_{\nu}(z) = J_{\nu}(z) + iY_{\nu}(z),$$

Eq. 25 can be simplified readily to

$$W^{(n)}_{k,m}(z) \sim \sqrt{\frac{\pi}{2}} f_1(n) |ns|^{\frac{1}{4}} |K(\sigma)|^{-\frac{1}{4}} e^{i\frac{\pi}{6}} H_{\frac{1}{3}}(ns). \tag{B-30}$$

All-Electric Heat Pump.—The glamorous, luxury-equipped *S.S. United States* may be queen of the seas, but she has to bow to a lowly barge on one score.

The blunt-nosed *Blind Pass*, out of the Gulf of Mexico, is fitted with science's latest answer to indoor temperature control—an all-electric heat pump developed by General Electric engineers. The ship's builder, Levingston Shipbuilding Corporation, of Orange, Texas, says she is the only ship afloat so equipped.

The heat pump's only fuel is electricity. The pump automatically extracts heat from cold winter air for warming the barge. Summertime cooling is accomplished by reversing the process. With one mythical blue-ribbon to its credit, the *Blind Pass* is willing to rest on its laurels. At present, the ship's builders have no plans for challenging the *S.S. United States'* speed record.

New Gamma-Ray Generator.—To help speed research into effects of dangerous and penetrating gamma rays on a variety of materials, a gamma-ray generator, designed specifically to give reproducible uniform total body irradiation, has been developed by the Navy. The new generator will allow ionizing radiation not readily obtained by X-ray equipment. The generator utilized a radio-active cobalt isotope known as cobalt-60 as a source of gamma rays. The generator theoretically can be used for five years with a loss of only 50 per cent of the radio-activity. Preliminary work was begun in 1949 by members of the Atomic Medical Research Institute.

Ductile Iron.—A cast iron that twists and bends was one of the features of the display of The International Nickel Company, Inc., at the Western Metals Exposition in Los Angeles, recently. The material will also bounce on impact instead of breaking.

Known as Ductile Iron, the material was developed by engineers of the company's Development and Research Division at its Laboratory in Bayonne, New Jersey, over a long period of time and was first made available commercially in 1949 when the year's total production was 3500 tons. This jumped to about 100,000 tons in 1952. Basically, it is gray iron to which has been added a small amount of magnesium and nickel. Industry has come to regard it as a brand new engineering material that bridges the gap between cast iron and steel.

Applications of Ductile Iron range from gun parts to plow points, and from crank shafts to heavy rolls for steel mills. It is wear and heat resistant as well as being ductile. Company research engineers have listed over 500 established and 200 experimental applications of the new material. Uses are found in almost every branch of industry from petroleum production and refining, to airplane engine building, and from pump construction to gear production. Both the U. S. Army and the U. S. Navy have authorized its use for equipment subject to battle damage for which ordinary gray cast iron in the past has been prohibited. It is used for many internal combustion engine parts—both automotive and marine.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

ELECTRONIC FLOWMETER

A new type of electronic flowmeter, capable of measuring the air currents in a still room or the rapid flow of fluids in pipes, has recently been developed at the National Bureau of Standards. This device, designed by Henry P. Kalmus of the NBS staff, utilizes the change in velocity of sound waves as a measure of fluid flow. It has a very fast response and does not obstruct the fluid currents to make the measurement. In addition, the signal-to-noise ratio is sufficiently high to permit the measurement of extremely small velocities.

Most of the present methods for measuring the velocity of flowing liquids depend on mechanical devices that usually introduce discontinuities in the fluid and have inherent inertia which prevents faithful reproduction of fast velocity changes. One type of flowmeter utilizes the principle of the homopolar machine; but this method cannot be used for gases, and the instrument is, at present, restricted to the measurement of conductive fluids. Another method makes use of the resistance change in hot wires placed in the path of fluid flow. However, besides presenting an obstruction to the flow, the hot wire is not generally applicable to fluids that would change chemically or physically as a result of the heat from the wires.

In the newly-developed flowmeter, a sound wave is transmitted over a fixed distance through the flowing fluid, and the phase of the received wave is compared with that of the transmitted wave. The sound energy is imparted to and taken from the fluid through the walls of the containing vessel. No part of the measuring system need come in direct contact with the medium under study. This characteristic of the system makes the unit applicable to the measurement of blood flow in the aorta or for the detection of the flow of chemicals in a closed system, for example, the coolant in a chain reactor. Minor modifications will permit the measurement of the flow of such fluids as gas or oil in a metal pipe, or a determination of the relative velocity of a ship to that of the water through which it is moving.

In instances other than those involving extremely high velocities, i.e., in wind tunnels, a simple system utilizing merely a stationary device to transmit energy into the medium and another stationary unit to receive it would be impractical because changes in distance or propagation velocity would produce errors far greater than the variations due to flow. For this reason, the transmitter and receiver of the new flowmeter are exchanged periodically without varying their locations.

* Communicated by the Director.

This is accomplished by using identical magnetostrictive or piezoelectric exciters, such as barium titanate crystals, as transmitter and receiver and by switching their connections alternately to the receiving and transmitting channel. Thus, the phasemeter displays two phase shifts alternately, one a function of the sound velocity plus the fluid flow and the other a function of the sound velocity minus the fluid flow. The difference between the two phase shifts is a measure of the velocity of the fluid. The switching can be made to operate faster than the inherent variations occurring in either the transmitter, receiver, or even the medium. Hence, the variations will occur at a minimum of one complete transmitter-receiver switching cycle, and the phase difference will remain unaffected.

In the first experimental model of the NBS flowmeter, a mechanical switching system operating at a 10-cycle-per-second rate is employed. The switching is achieved by a commutator that is inserted between the crystals on one side and a 100-kc oscillator and receiver on the other. The oscillator is connected to an amplifier containing carefully designed limiters that produce a square wave voltage to be injected on one grid of a multigrid phasemeter tube. The receiver amplifier also furnishes a limited signal to the other grid of the phasemeter tube.

The phasemeter tube is by-passed by a capacitor so that fluctuations at the sound wave frequency are removed and the fluctuations at the switching rate may be applied to a synchronized rectifier. The rectifier is actuated in synchronism with the commutator, but its active period is shorter so that harmful transients occurring during the crystal transition are suppressed.

A simple commutator, one that would merely switch the crystals alternately to the receiving and transmitting channels, is unsuitable because of the capacitive cross-coupling between the two channels. In order for the transmitted and received pulses to be absolutely representative of the separation between the crystals and the velocity of the fluid, there must be a minimum of coupling between the two channels. By very careful design of the commutator, the leakage capacitance can be reduced to about 5×10^{-5} $\mu\mu$f., and much of the fringe effects and ground currents is eliminated.

The rotor of the commutator is made in the form of a sandwich, consisting of two pieces of insulating material fastened to a grounded conductive shield. Two conductive segments or sectors are mounted to the insulating material but with no connection to the shield. The shield is brought out to the maximum diameter of the rotor, thereby reducing the fringe effects between the sectors. Four rows of carbon brushes are arranged so that the crystals are connected to either the receiver or oscillator for all positions of the rotor. Capacitive coupling between the sectors is reduced by maintaining a clearance of less than 0.015 in. between the sector shield and the rotor. Similarly, close-

fitting shields are provided at the ends of the rotor to assist in reducing fringe effects.

The output of the phasemeter is applied to an electronic high-impedence voltmeter (which can be calibrated directly in terms of velocity) through the rectifier-switch geared to the commutator drive. Fast velocity changes can be observed on the screen of a cathode-ray oscilloscope connected to the plate of the phasemeter tube. Thus, if the plate voltage is made visible, the difference between the voltages developed "upstream" and "downstream" can be used directly to estimate the flow-velocity instantaneously.

A modification to the commutation operation is the use of an electronic switch. By using vacuum tubes, the switching frequency can be increased to 1000 cycles, for example, and the time of response can be correspondingly shortened. Four pentode amplifiers are placed between the crystals on one side and the oscillator and receiver on the other. The screen-grid supply is derived from a square-wave generator with a frequency equal to the switching rate (1000 cps). When the screen voltage is positive, the pentode acts as a normal amplifier; when the potential becomes negative, the electron stream in the tube is blocked and good shielding is provided. The amplifiers are so arranged that one pair operates at a time during each half of the switching cycle.

Further modifications in the flowmeter are being considered; one in particular is concerned with flow in metal pipes. A pulsed and gated phase comparison system will be used for this purpose to eliminate the effects of the direct transmission of the sound through the wall of the pipe.

DETERMINING ORIENTATION OF METAL CRYSTALS WITH POLARIZED LIGHT

Knowledge of the orientation of crystals of metals is often of practical importance, but no generally satisfactory method has been available for determining the orientation of the small crystals found in commercial metals. Because of the need for such a method, the National Bureau of Standards is investigating the possibility of estimating crystal orientation from measurements of changes in the characteristics of polarized light. The work is being conducted by H. C. Vacher of the Bureau's mechanical metallurgy laboratory. Results so far indicate that, in the case of the industrially important alloy of 70 nickel and 30 copper (Monel), crystal orientation can be correlated with extinctions of polarized light.

Commercial metals are composed of aggregates of small crystals or grains. The magnitude of many important properties—such as strength, elasticity, and magnetism—differs in different directions in these small crystalls. In some specimens of some metals the orientations of the individual crystals do not follow a random distribution; instead, a "preferred orientation" predominates, and the overall

properties of the metal vary with direction. Also, the orientations of adjacent crystals influence their interactions when the aggregate is subjected to strain, heat, or corrosive environment.

X-ray diffraction methods of determining crystal orientation have been attempted, but cannot be applied conveniently to small crystals. The metallurgical microscope is convenient for observing small crystals, but its use is generally limited to showing differences in orientation qualitatively. In recent years, these microscopes have been equipped so that the specimen can be illuminated with plane-polarized light. When a metallographically polished and optically active surface is examined with such an arrangement, characteristics of reflected polarized light change measurably from crystal to crystal.

In the commercial metallographic microscope used for the NBS study, the analyzer is fixed in the crossed position with respect to the polarizer, and the light irradiates the polished surface of the specimen at normal incidence. Under these conditions, when an optically anisotropic surface is rotated 360 degrees around the axis of the incident beam, four minima (extinctions) and four maxima will alternately appear in the intensity of the reflected light.

Although metals having a cubic crystal structure are not inherently optically anisotropic, the surfaces of many such metals—including Monel—become optically anisotropic after etching with conventional metallographic reagents. The angular positions of the polarized-light extinctions of Monel differ for adjacent crystals, and it was found that these positions remained the same after repolishing and re-etching. It seemed to follow that the apparent optical anisotropy, being thus independent of the etching treatment, could probably be correlated with crystal orientation.

Examination of annealed Monel specimens showed clusters of crystals in which each crystal was adjacent to at least one annealing twin. (A twin relationship is indicated in a polished and etched surface when two crystals are separated by a straight line.) It was also observed that some of the clusters contained a crystal whose straight-line boundaries were in four different directions. This made it possible to determine the orientation of that crystal from geometry and a knowledge of the angular relationships of the twinning planes and the coordinate axes of the crystal. In the Monel crystal, the twinning planes and coordinate axes are the four octahedral planes and the three cubic axes respectively. After determination of the orientation of one crystal it was possible to determine the orientations of the twinned crystals. The orientations of 12 crystals were determined and their extinction positions measured.

Polarized light reflected from polished and etched surfaces that were parallel to a cubic plane or an octahedral plane changed but little in intensity with rotation of the specimen. Surfaces of other orientations,

however, gave extinctions whose positions could be measured with a fair degree of accuracy. Results showed that one of the four extinctions was near the cubic pole which was inclined to the surface at the smallest angle.

The NBS study also indicates that the optical anisotropy exhibited by Monel is caused by parallel furrows in the surface and not, as has sometimes been suggested, by a film. The furrows are apparently formed by the tendency of the etching reagent to produce etch pits whose facets are parallel to cubic planes.

NOTE: For further details of the NBS study, see "A Correlation of Polarized Light Extinctions, with Orientation in 70 Nickel—30 Copper Alloy," by H. C. Vacher, *J. Research NBS*, Vol. 49, p. 149 (Sept. 1952); reprinted as NBS Research Paper 2351, 10 cents from Superintendent of Documents, Government Printing Office, Washington 25, D. C.

BUTYL-IMPREGNATED LEATHER

The process for impregnation of leather with natural rubber, which the National Bureau of Standards developed in 1949, has recently been extended to permit the use of polyisobutylene[1] as an impregnant. Soles impregnated with this butyl polymer have the same greatly improved abrasion and water resistance as those containing rubber. However, polyisobutylene has a distinct advantage over rubber as an impregnant in that it eliminates the milling operation required to reduce the molecular size of natural rubber sufficiently to allow deep penetration. It also results in large savings of time, labor, and materials in the tannery. The butyl-impregnation process was developed by René Oehler, Sverre Dahl, and T. J. Kilduff of the NBS leather laboratory.

Impregnation with the butyl compound merely requires soaking of the naturally porous leather in a solution of the polymer. The impregnating solution is prepared by dissolving polyisobutylene, having a molecular weight range between 50,000 and 100,000, in common solvents such as gasoline, kerosene, benzene, or chloroform. The leather is air-dried and immersed in the solution for about three hours. It is then removed from the solution, the excess solvent is wiped off, and the remaining solvent is evaporated from the leather.

If vegetable-tanned crust leather is used and the grain or hair side of the leather is split off to a depth of about 40 mils before impregnation, penetration by the polyisobutylene is more rapid and uniform. This results in greater abrasion resistance, and the top grain split becomes a valuable by-product for use in luggage, upholstery, bookbinding, and other fine-leather applications. Because vegetable-tanned crust leather requires less tanning and processing than commercial sole leather, the butyl-impregnation process makes possible considerable savings not only in time and labor but also in expensive tanning materials, which at present are almost exclusively imported.

[1] For further details, see "Treatment of Leather with Polyisobutylene," by René Oehler, Sverre Dahl, and Timothy J. Kilduff, *J. Am. Leather Chem. Assoc.*, Vol. 47, p. 642 (1952).

NBS studies have shown that the butyl polymer is well suited for treating shoe soles and upper leather, as it reduces water absorption by about half and doubles abrasion resistance. Service tests were carried out in which the same individual wore one sole of butyl-treated vegetable-tanned crust leather mated with a control sole of untreated leather obtained from the same part of the animal, and finished as commerical sole leather. Cross-mating of these matched soles eliminated any bias due to differences between right and left feet of the test personnel. Of the 12 pairs of soles tested, the control sole wore through before the butyl-impregnated sole in every case. The tests showed that the treatment increases wear by about 80 percent.

Research in materials and techniques for leather impregnation is continuing at NBS. The Bureau is also carrying on pilot-plant studies of butyl-impregnation processes under the sponsorship of the Department of the Navy.

THE FRANKLIN INSTITUTE

MINUTES OF STATED MEETING

March 18, 1953

The Stated Monthly Meeting of The Franklin Institute was held on March 18, 1953, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. Approximately 390 persons attended.

The President stated that the Minutes of the Annual Meeting in January were printed in full in the February JOURNAL. There being no corrections or additions, they were approved as printed. He stated further that the Minutes of the Stated Meeting in February were being printed in the March JOURNAL, which had not yet been mailed to members. Approval, therefore, was postponed until the next meeting.

The Secretary reported the following elections to membership for the month of February;

Active	27
Associate	26
Student	13

and a total membership of 6636 as of February 28, 1953.

The Secretary also reported that the National Asphalt Research Center, a project of The Franklin Institute Laboratories for Research and Development, held its third annual meeting at the Institute on February 16th and 17th. During the past year, NARC sponsors increased from twenty-six to twenty-nine. Fifty-seven representatives attended the meeting.

The President welcomed the officers and members of the Rittenhouse Astronomical Society and presented its President, John W. Streeter, who is also Assistant Director of the Fels Planetarium of The Franklin Institute. Mr. Streeter spoke of the pleasant association the Society has enjoyed with the Institute during the past twenty years and of the encouragement given the Society by the Institute. In recognition, the Society presented a bronze replica of the Rittenhouse Medal to the President of the Institute. Dr. I. M. Levitt, Director of the Fels Planetarium, made the presentation.

Mr. Streeter introduced Dr. Harlow Shapley, Paine Professor of Astronomy at Harvard University, to whom the Society awarded the silver Rittenhouse Medal. In his lecture, "The Material Universe, 1953 Model," Dr. Shapley brought the audience up to date on matter, radiation, space, and time. He also spoke most interestingly of the investigations presently underway concerning, among other things, the anticenter of the galaxy, the corona of stars around the galaxy, and the structure of the center of the galaxy. Dr. Shapley's lecture, excellently illustrated and sparked with humor, was very favorably received by the audience.

The meeting adjourned at 9:50 P.M. with a rising vote of thanks to Dr. Shapley.

HENRY B. ALLEN
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

ZADEH, L. A.: A Contribution to the Theory of Nonlinear Systems.

LAUER, HENRI: Operating Modes of a Servomechanism with Nonlinear Friction.

MOON, PARRY, AND DOMINA EBERLE SPENCER: Some Coordinate Systems Associated with Elliptic Functions.

COURT, ARNOLD: Wind Extremes as Design Factors.

ALDRICH, LELAND E.: Solution of Algebraic Equations.

MINORSKY, N.: On Interaction of Nonlinear Oscillators.

MUSEUM

The Franklin Institute has always had a sympathetic concern for young people with scientific and mechnical inclinations, and has tried to assist in solving their problems for instruction and entertainment. Whatever steps have been taken in this direction, they have usually been enthusiastically received. One of the most recent activities arranged for the benefit of young people is a program centered upon the Museum and called "Science Adventures." This program, tried experimentally in 1949 as a series of especially guided tours of the Museum, has gradually evolved into an integrated series of workshop activities and lecture-demonstrations. Last year, when the new program was announced, it provoked so much interest that the subsequent enrollment of students exceeded anticipations to such an extent that the younger students had to be formed into two groups.

The older students (12 to 15 years) devote most of their time to work in the Museum, where the members of the staff give demonstrations with the exhibits, supplemented with additional material where necessary. This opportunity to see and handle the material with more intimacy than the Museum display permits and to have the essential pieces described in detail is much enjoyed by the students. It provides a rewarding and instructive experience for the staff, also. The indoor work is supplemented with field trips to laboratories or industrial plants, where the greatest courtesy has been extended the young visitors.

Work with younger students is always rather more difficult because of the less developed intelligence, but a happy solution to the difficulties has been found in reducing the amount of lecturing in order to concentrate upon interesting activities in which the students play the active part. This group covers an age spread between 8 and 12 years. By basing their work on workshop activity, combined with games in which elementary scientific principles are employed, no difficulty has been experienced in holding the students' interests while imparting instruction. The subjects covered last year embraced clocks; oxygen and oxidization; dry cells, batteries, and electro-magnets; magnetism; and kitchen chemistry. During the course of their meetings (which are held twice a week for a month) the young student performed experiments, learned how to perform numerous scientific tricks, and played several games with the knowledge he had acquired. He learned how to make a fire-extinguisher, a compass, an electro-magnet, and other things.

The course is intended to introduce young people to science, and taken altogether each of the eight meetings each student attends proves to be a real adventure in science. Now that the summer months are approaching and the question of a child's vacation may come up for discussion, it would be wise to consider the prospect of "Science Adventures." Particulars may be obtained from the Educational Department of the Museum and, in view of the rapid and extensive enrollment last year, an early inquiry for information will forestall disappointment.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

GIFT OF FIRST EDITION OF EUCLID'S "ELEMENTS"

It is not often that a library is able to announce the acquisition of a book as notable for both historical importance and typographical excellence as the first edition of Euclid's *Elements of Geometry*. It is therefore with special pleasure that the Library Committee reports the gift by Mr. Boies Penrose to the rare book collection of the Franklin Institute Library of a fine copy of this outstanding work.

For over twenty-two hundred years Euclid's *Elements* has been the foundation-stone of mathematics and the basis for the development of the sciences of astronomy, navigation and mechanics. The book has been described as the most famous in the history of science. Sir Thomas Heath (in *The Legacy of Greece*, 1922) truly says of it that "*No work presumably, except the Bible, has had such a reign.*"

Prior to the invention of printing, Euclid's work circulated among scholars in manuscript. The original Greek text was early translated into Arabic, and in the 13th century a Latin translation from the Arabic was made by the English philosopher Adelard of Bath. This translation, annotated by Campanus of Novara, was used for the first printed edition of the book which appeared in 1482 at Venice. It was not until 1533 that the Greek text was first printed by Johann Herwagen at Basel. Since 1482 thousands of editions of the *Elements* have appeared throughout the world, none, however, in as distinguished a format as the first edition which came from the press of Erhard Ratdolt of Venice on the 25th of May, 1482.

Erhard Ratdolt was born in Augsburg, the son of a carpenter. He commenced printing in Venice in 1476 in association with Peter Löslein, supposed to have been a business man, and with Bernard Maler, whose skill as a wood engraver was responsible for the exceptionally fine ornamental borders and initial letters which characterized many of Ratdolt's subsequent publications. The partnership lasted for only two years, but Ratdolt continued in the business in Venice until 1484, when he was recalled to Augsburg by its Bishop to become a printer of diocesan service books and scientific works in his native town.

Ratdolt's books have been held in the highest esteem for their fine typography. His edition of Euclid is a small folio printed on 138 leaves in gothic type, 45 lines to the page, with woodcut initials throughout and a beautiful three-side woodcut border on the first page of the text. The geometrical diagrams, numbering over 600, are printed in the margins. They are of great technical interest, being, according to Ratdolt himself, the first extensive set of such illustrations to appear in a printed book. One admirer[1] of Ratdolt's work on the 1482 Euclid was moved to say:

> In contemplating the finished perfection of his work, the work of an artist who could make of the diagrams set in the wide margins of his book an added beauty to it, it is difficult to remember that it was also the work of a pioneer. As an explorer may hew a way through the virgin forest and thereby determine the course of a road which will become permanent, so the great printer found a style and laid down a method which were followed by his successors for centuries.

[1] CHARLES THOMAS-STANFORD, "Early Editions of Euclid's Elements," London, Bibliographical Society, illustrated Monograph No. XX, 1926.

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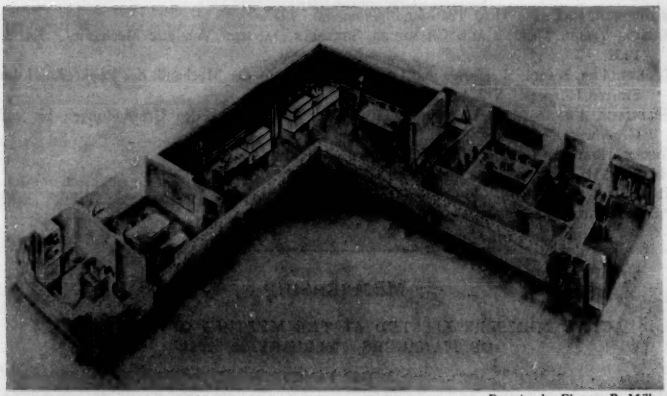
THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

AN A-C NETWORK ANALYZER

Power companies in this area are extending their networks—generating more power and distributing it to more places—to supply the electricity that is so vital to homes, farms and industries. Many complex engineering problems are involved in meeting present requirements efficiently and preparing for future expansion. Out of this need for growth and planning has come another cooperatively sponsored project for our Laboratories for Research and Development.



Drawing by Eleanor B. Miller

Artist's conception of the network analyzer installation at The Franklin Institute. The building contains, left to right: spaces for air-conditioning equipment and power supplying M-G set; conference room; smaller section of the analyzer; larger section (sliding partitition assures privacy if both units are operated simultaneously by different companies); conference room; office area; and foyer. The entire building is windowless, illumination being provided by special fixtures that eliminate glare on panels and working areas.

Seven of these power companies are installing an A-C Network Analyzer in these Laboratories to study existing or proposed power systems. A miniature power system scaled down to 1/100,000, the analyzer has electric circuits that represent generators, transmission lines, transformers and loads. In minutes, or hours at most, the operators of the analyzer can connect these elements in imitation of any given power system to discover how the full-scale system works (or will work, if it

has not yet been built), whether or not it will be economic, and how it might be modified to give better service. With the analyzer, nearly all problems of designing and operating a power system can be solved: transient and steady-state stability, load control, voltage regulation, relay application, etc.

A company wishing to analyze an existing or proposed power system will bring the problem to The Institute in the form of a single-line diagram (similar to a block diagram) with appropriate values for the system's components. Impedances in the system are expressed in ohms, or in per cent on a kva. base. Lines, transformers, current-limiting reactors and the internal impedances of generators are repre-

Courtesy of Westinghouse Electric Corp.

Portion of a typical analyzer installation, showing operator's console.

sented by resistance and reactance circuits, loads by shunt impedances. Capacitors represent synchronous or static condensers. Single-phase power sources, with both voltage and phase angle separately adjustable, represent generators.

The various circuits of the analyzer are connected to form a suitably scaled model of the power system in question. The master instruments —a voltmeter, ammeter, and watt-varmeter—are used to meter the various circuits of the model system. Multipliers reverse the scale of the analyzer and allow the meters to give actual values of current,

voltage and power. The diagram of the system is mounted on a special board placed near the instrument panel. Beneath the translucent top of this board are 5500 positions for small lights that indicate to the operator which circuit is being metered. This makes the operation both rapid and accurate.

The analyzer will incorporate 580 circuits in two separate units, one having one-third, the other two-thirds of the total capacity of the device. Each unit can operate separately, or both together if the problem demands. The analyzer, conference rooms, office, storage space, power supply and air-conditioning machinery will occupy 2900 square feet. An operator at either one of the two instrument control desks (one with each unit) will be able to make measurements throughout the entire analyzer.

Built by the Westinghouse Corporation, the $400,000 analyzer will be housed in a special structure atop The Franklin Institute. In mid-1954, when installation and tests are complete, the analyzer will be put to work. The participating firms will share the cost of operating the instrument in proportion to the use they make of it. Should they not require all their allotted time, they are free to relinquish some of it to other utilities.

The Laboratories welcome the opportunity of helping these companies maintain the service they render to industry and the public, and plan for new facilities to keep pace with the growth of the area.

B. B. YOUNG

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

CYTOPLASMIC LOCALIZATION OF HIGHLY POLYMERIZED LIVER RIBONUCLEIC ACID

BY

EDWARD L. GRINNAN AND WILLIAM A. MOSHER

Highly polymerized ribonucleic acid (RNA) was isolated from cytoplasmic extracts of rat and calf liver (1). In view of the morphological and biochemical heterogeneity of cytoplasm it was desired to establish the intracellular localization of the RNA. Cytoplasmic fractionation was carried out by differential centrifugation of sucrose homogenates of rat liver, and RNA was isolated from three fractions: mitochondria, microsomes, and supernatant. Nearly all the highly polymerized RNA of the cytoplasm was found in the microsome fraction.

EXPERIMENTAL

Livers from young male Wistar-strain rats of 150 to 200 gm. weight, fed a normal diet of Fox Food Blox and water *ad libitum*, were used. The animals were decapitated and allowed to bleed freely. Immediately after excision, the livers were frozen in dry ice-acetone mixture 10 to 20 min.; livers not used immediately were stored under dry ice refrigeration. Temperatures of 0° to 8° C. were maintained during the preparation and fractionation of the homogenates in order to minimize ribonuclease activity.

An International type SB, size 1 model centrifuge, equipped with an angle head containing lusteroid tubes of 100-ml. capacity, was used to sediment nuclei and mitochondria. A preparative ultracentrifuge equipped with a cooling system was used to sediment microsomes.

The livers in 100-gm. lots were passed while in a semi-frozen state through a meat grinder into 2.5 volumes of 0.25 molar sucrose, except for preparation 1 in which the sucrose used was 0.88 molar as advocated by Hogeboom, Schneider, and Pallade (2). The liver suspension was homogenized in a Waring Blendor 3 times for 10 seconds each time, and the homogenate stirred at relatively low speed for 15 min. The suspension was centrifuged at 3,000 times gravity (calculated for the bottom of the tubes) for 50 min. to bring down a mixed sediment of whole cells, nuclei, and mitochondria; the sediment was suspended in 0.25 molar sucrose solution and centrifuged 3 times at 3,000 times gravity for 5 min. each time in order to remove whole cells and nuclei. The supernatant from the last centrifugation was centrifuged at 3,000

times gravity for 50 min. to sediment mitochondria. The mitochondrial sediment was suspended in physiological saline and washed once by centrifugation at 3,000 times gravity for 30 min. The mitochondrial fraction thus obtained was relatively free of nuclei as determined by microscopic examination, and when suspended in 0.88 molar sucrose, the mitochondria were morphologically similar to those reported previously (2).

The supernatant from the initial centrifugation which removed whole cells, nuclei, and mitochondria, was centrifuged at 100,000 times gravity calculated for the bottom of the tubes (70,000 times gravity for the center of the tubes) for 60 min. to sediment microsomes. The clear pink supernatant was readily removed by decantation from the cherry-red gelatinous microsomal pellet; the microsome fraction was dispersed in 0.25 molar sucrose prior to the isolation of RNA.

One of the microsome fractions, preparation 4, however, was not obtained by ultracentrifugation; the supernatant after the initial centrifugation at 3,000 times gravity for 50 min. was adjusted to 0.13 per cent $CaCl_2$ with stirring. The extract was placed in the refrigerator for 15 min. to allow agglutination of microsomes (3, 4). The aggregated microsomes were then sedimented in the International centrifuge at 3,000 times gravity for 30 min. The microsomes thus sedimented consisted of a buff, amorphous sediment not tightly packed. The supernatant was drawn off to within 1 cm. of the microsome layer.

RNA was isolated from both the microsome and supernatant fractions by the guanidine hydrochloride method previously described (1). Yields, N/P ratios, and S_{20} of the preparations of microsomal and supernatant RNA are given in Table I. In general, the N/P ratios

TABLE I.—*RNA from Microsomal and Supernatant Fractions of Rat Liver Isolated by Guanidine Hydrochloride Method.*

	Microsomal					Supernatant				
Preparation No.	Yield, mg./100 gm. Liver	N, %	P, %	N/P	S_{20}	Yield, mg./100 gm. Liver	N, %	P, %	N/P	S_{20}
1*	170	15.6	7.9	1.98	6.1	37	17.4	7.2	2.42	6.0
2	210	16.9	7.9	2.14	6.2	9	—	—	2.20	7.0
3	205	17.6	7.4	2.38	6.9	8	—	—	2.78	6.1
4**	125	18.4	7.4	2.49	6.4	48	18.4	7.0	2.63	6.2
Average					6.4					6.3
5 Butter Yellow	72	15.6	7.4	2.11	6.1	<5	—	—	—	—
6 Control	100	16.0	8.0	2.00	6.4	<5	—	—	—	—

* 0.88 Molar sucrose medium.
** Calcium chloride precipitation of microsome fraction.

were somewhat higher than those reported previously for preparations from unfractionated extracts; despite the higher N/P ratios, biuret tests for protein yielded negative results, and the preparations sedimented as a single homogeneous, though polydisperse component in the

analytical ultracentrifuge. Nitrogen analyses were made by the micro-Kjeldahl method, and phosphorus was determined by a modification of the Fiske-Subbarow method. Sedimentation velocity constants (S_{20}) were determined with a Beams type air-driven ultracentrifuge at a speed of 42,000 rpm. (*ca.* 150,000 times gravity). The sedimentation runs were carried out in 0.07 molar NaCl, pH 6.7 phosphate buffer, at an RNA concentration of 0.25 per cent. The S_{20} determined at this concentration was only slightly lower than that obtained by extrapolation to zero concentration (1).

Mitochondrial RNA could not be isolated by the guanidine hydrochloride method; therefore, a more drastic method was used. The mitochondrial fraction was suspended in 10 per cent NaCl solution and heated in a boiling water bath for either 15 or 30 min., the suspension was cooled and centrifuged, and the sediment washed once with hot 10 per cent NaCl and centrifuged. The combined supernatants were added to 3 volumes of cold 95 per cent ethanol, and the gelatinuous precipitate of RNA was centrifuged. The RNA was dissolved in water and deproteinated by the method of Sevag *et al.* (5) by shaking with chloroform-butanol (4:1). The purified mitochondrial RNA was recovered by ethanol precipitation, dissolved in water and lyophilized. Yields, N/P ratios, and S_{20} of four preparations of mitochondrial RNA are given in Table II.

TABLE II.—*RNA* from Mitochondrial Fraction of Rat Liver.*

Preparation No.	Yield, mg./ 100 gm. Liver	Per Cent of Dry Fraction	N, %	P, %	N/P	S_{20}
3**	25	—	13.2	7.6	1.72	3.0
4**	23	3.0	12.4	7.4	1.68	2.7
5*** Butter Yellow	28	3.0	15.4	9.2	1.68	3.7
6*** Control	22	3.3	13.8	8.0	1.73	3.8

* Isolated by heat denaturation of mitochondria in 10 per cent NaCl at 100°.
** Heated in boiling water bath for 30 min.
*** Heated in boiling water bath for 15 min.

To investigate the effect of butter yellow (*p*-dimethylaminoazobenzene) on the distribution and particle size of liver RNA, livers of rats fed butter yellow over a period of 5 months and which were in the pre-hepatoma stage of butter yellow feeding were used for preparation 5; preparation 6 was the control preparation from the livers of normal rats placed on the Fox Food Blox diet 5 months prior to use. Previous experiments indicated that in the isolation of RNA by the guanidine hydrochloride method (1), the heat denaturation at 100°C. could be omitted as a non-essential step; therefore, in preparations 5 and 6 the washed nucleic acid-protein complex was dissociated by shaking with chloroform-butanol only. Although there was no apparent

difference between the particle size of microsomal RNA from normal livers and from the livers of rats fed butter yellow, the yield of RNA was considerably less in the latter.

DISCUSSION

It seems evident that nearly all the highly polymerized RNA of the cytoplasm of normal liver cells is located in the microsomes. Only traces of RNA were recovered from the supernatant fraction, in agreement with the work of Brachet and Jeener (6) who reported that in adult organs only trace amounts of RNA are present in the supernatant fraction. The RNA recovered from the supernatant fractions was in all probability microsomal RNA which did not sediment in the microsome fraction. Because of the higher density 0.88 molar sucrose medium in preparation 1, and the calcium chloride fractionation in preparation 4, the microsomes apparently were separated into heavy and light particles which appeared in the microsomal and supernatant fractions respectively.

Although no consistent difference was found between the sedimentation constants of RNA isolated from the microsomal and supernatant fractions, the average sedimentation constant (6.4) was about 30 per cent lower than the average value for 15 preparations isolated directly from unfractioned cytoplasmic extracts of liver (7). The lower sedimentation constants are assumed to be the result of ribonuclease activity during the rather extended period of fractionation; these sedimentation constants indicate a molecular weight in the order of 150,000, approximately half that of preparations isolated directly from unfractionated cytoplasmic extracts (1, 7). An attempt to inhibit ribonuclease activity by the use of 0.3 per cent formaldehyde (8, 9) in the homogenates was unsuccessful as the formaldehyde interfered with the precipitation of the nucleic acid-protein complex from the 4 M guanidine hydrochloride solution.

The inability to isolate mitochondrial RNA by the guanidine hydrochloride method suggests differences in the manner in which RNA is bound to the mitochondria and to the microsomes. Despite the somewhat drastic method used in the isolation of mitochondrial RNA, the sedimentation constants of preparations 5 and 6 which were subjected to only 15 min. heating in the boiling water bath are indicative of a molecular weight in the order of 50,000. The N/P ratios of mitochondrial RNA were significantly lower than those of microsomal RNA although the same method of protein removal was used for both. The low N/P ratios of the mitochondrial RNA may have been the result of the more drastic method of isolation; the high N/P ratios of microsomal RNA could be caused either by the presence of some non-nucleic acid nitrogen, or to a deficiency of phosphorus in the highly polymeric chain as a result of sharing of phosphoric acid among three instead of between two nucleosides.

The absence of a specific RNA in the supernatant fraction, and the inability to isolate mitochondrial RNA by the guanidine hydrochloride method allows this method to be used for the isolation of microsomal RNA directly from an unfractionated cytoplasmic extract of normal liver. It may be concluded that the preparations reported previously (1) consist only of microsomal RNA. In view of the reported RNA shift from the microsome to the supernatant fraction of tumor and embryonic cells (6, 10, 11), the investigation of the cytoplasmic RNA of hepatoma and embryonic liver in regard to distribution and particle size would be highly desirable.

SUMMARY

Highly polymerized ribonucleic acid (RNA) of rat liver cytoplasm was found to be localized almost exclusively in the microsomes. Only traces of RNA were obtained from the supernatant fraction after centrifugation of the microsomes at 100,000 times gravity. The average sedimentation velocity constant, 6.4, of the microsomal RNA was about 30 per cent lower than that of RNA isolated directly from unfractionated cytoplasmic extracts, probably as a result of ribonuclease activity during the fractionation procedure. RNA could not be isolated from mitochondria by the guanidine hydrochloride method used for the isolation of microsomal RNA; a more drastic method yielded mitochondrial RNA with an S_{20} of 3.7. The yield corresponded to a content in mitochondria of at least 3.3 per cent RNA.

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"Supersonic" Eyes.—Pilots of the future who fly planes and rockets at 1,800 miles an hour will have to develop appropriate supersonic eyes to go with them or suffer the consequences. Col. Victor A. Byrnes of the Air Force School of Medicine recently presented this supersonic eye theory to the Institute of Aeronautical Sciences in New York.

He pointed out that a sharp-eyed pilot emerging from a cloud at 1,800 miles an hour might spot an object, and in practically the same instant he saw and became conscious of it, his plane would have traveled three-quarters of a mile. He said two such planes approaching 500 feet apart could crash without the pilots ever having seen anything. Another problem could be the distortion of view caused by the build-up of a compression of air waves that would cause the pilot to misjudge the true location of an object.

Col. Byrnes is quoted as saying that this eye problem is beyond the realm of science to provide the answer.

But is it? National Patent Council agrees that human physiology may require ages of evolution to grow pilot's eyes adequate for this kind of flying. Human vision, as nature provides it, is comparatively fragile and severely limited to the expanded requirements of this modern age.

Long ago science discovered the use of optical glass to aid natural vision. Improvement through the years, induced by the protection afforded through our Patent System, have produced lenses to enable the eyes to observe minute organisms, otherwise invisible to the most perfect of natural direct eyesight. Other lenses have enabled the natural eyes to study the substances of distant planets and stars. Inventions continue into this supersonic age.

The magic computer has been greatly reduced in size and weight. Radar extends human vision through fog and darkness. Who can tell but some future inventor will meet the problem which Col. Byrnes believes to be beyond the aid of science? With the incentive provided by our Patent System, it seems likely that some inventor may find a way to wed the computer and the radar into a device that will, with electronic speed, see and automatically actuate the controls where the pilot's eye, brain, and hand could never possibly avoid disaster.

High voltage fluorography, combined with rapid automatic film processing, may show the way to lower cost and faster techniques for the production inspection of melt-loaded ammunition in ordnance plants. Key to the success of the new technique lies in the discovery that fluorescent screens are capable under certain conditions of a highly discriminating brightness-response at radiation intensities on the order of two million electron volts, even though this energy is far above their usual peak response energy. The proposed fluorographic technique is expected to supplement the present methods of X-ray inspection.

BOOK REVIEWS

VOLUMETRIC AND PHASE BEHAVIOR OF OIL FIELD HYDROCARBON SYSTEMS, by M. B. Standing. 123 pages, diagrams, 22 × 29 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

The petroleum engineer and the geophysicist will find this book a source of technical information that cannot be found elsewhere. Dr. Standing has surveyed the scattered literature of the past twenty years pertaining to the phase behavior of hydrocarbon systems found in petroleum reservoirs. In this volume he has successfully shown the relationship between laboratory test data on the phase behavior of these gas-condensate systems and actual conditions in the reservoir.

There are eight chapters: (1) Introductory Phase Behavior and Fluid Flow Concepts; (2) Sampling Methods and Apparatus; (3) Behavior of Gases; (4) Behavior of Liquids; (5) Vapor-Liquid Equilibria; (6) Gas Condensate Systems; (7) Dissolved Gas Systems; and (8) Material Balances. Each chapter is followed by an excellent summary of its principal points. The appendices contain a glossary of technical terms, a list of mathematical symbols, and a list of pertinent literature references.

Compactness is one of the most striking features of this book. The author covers the subject thoroughly in 108 pages without omitting any important points from his discussion. He achieves this compactness by (*a*) using an outline form throughout the book, (*b*) presenting many data in summary graphs and tables, and (*c*) suppressing details of experimental techniques and instruments that are already familiar to the audience to which the book is directed.

This is an extremely valuable book to the field engineer since it provides the means of using laboratory data to predict the behavior of hydrocarbons in the petroleum reservoir. It also provides calculation charts from which the engineer can estimate fluid properties and material balances from his own field measurements. The various well-sampling techniques are critically evaluated, and guidance is offered as to which applies best to a given set of reservoir conditions. From this partial list of contents, it is obvious that no petroleum engineer, whether in the laboratory or the field, can afford to be without Dr. Standing's excellent book.

D. H. RUSSELL

NUCLEAR STABILITY RULES, by N. Feather. 162 pages, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1952. Price, $4.00.

This small monograph is a clear, concise compilation, correlation and critical evaluation of data in the field of nuclear disintegration. As such, the title could just as well have been "Nuclear Instability Rules." These data are lucidly presented with the aid of 27 diagrams and with a view to discovering the regularities in nuclear stability. There is little effort made to understand these regularities. One of the aims of the author has been to so "marshall the experimental facts that the theorist is most likely to be inspired by valid—even 'correct' ideas on contemplating them." Professor Feather worked five years on this monograph and the clarity and completeness of his presentation is testimony to his successful effort. The mass of experimental data (200 references) is discussed with reference to recent theory. The author has performed a valuable service in making this latest comprehensive survey in the field of stability. Not only has he made original contributions to the field in the past, but he [illegible] new conclusions in the light of recent data.

The monograph opens in fine logical fashion with an extended empirical definition of stability. Stability is not a yes-or-no proposition; there is always the possibility of an unobserved disintegration either because of extremely low activity or because the decay product is unobservable. There are chapters on the systematics of stable nuclei, regularities in α-disintegration, regularities in β-disintegration, and a final brief chapter on spontaneous fission and the number of elements. It is shown that fission becomes more and more rapid as the

atomic number Z is increased and that this fact places a natural upper limit for Z of about 110 upon nuclear synthesis.

Experimentally determined relations between mass number, atomic number, type of radioactivity, decay constant, disintegration energy, parity, nuclear spin and degree of forbiddenness comprise the major portion of the book. Use is made of revised Geiger-Nuthall and extended Sargent diagrams. Deviations from the observed regularities are discussed. These regularities in stability as a function of atomic number and atomic mass lead to "magic" numbers in nuclear structure. No explanation for the existence of these "magic" numbers is offered.

The author's style is, in the main, quite excellent and it makes for good readability. However, the author has a tendency to use extremely long and complicated sentences which run to over ten lines. Then, too, there is the use of parentheses within parentheses, a practice which may lead to confusion. Another bad practice is the interjection of references in the middle of sentences; this tends to interrupt the train of thought. The author uses "in respect of" when we would write "with respect to." On the whole, these minor defects are of little weight compared with the generally excellent presentation.

Only a basic knowledge of nuclear physics is needed for an intelligent understanding of *Nuclear Stability Rules*. LEONARD MULDAWER

MECHANICS. LECTURES ON THEORETICAL PHYSICS, VOLUME I, by Arnold Sommerfeld; translated by Martin O. Stern. 289 pages, diagrams, 16 × 24 cm. New York, Academic Press, Inc., 1952. Price, $6.50.

The active scientific life of Arnold Sommerfeld was unusually long, spanning nearly sixty years of research and teaching. These were the eventful years during which physics made the profound transition from classical to quantum methods. Sommerfeld's technical contributions to the transition are well known through his publications; his contributions as a teacher, sustained throughout his long career, are less tangible, but one need only note the number of his former students included in the ranks of outstanding physicists in this country alone to realize how powerful his influence has been.

But these were postgraduate and post-doctorate students, drawn to Munich by Sommerfeld's reputation in advanced physics. Of Sommerfeld's teaching of the basic classical physics, which is the foundation of a physicist's education, there has been little evidence in this country. He did not prepare his Lectures on Theoretical Physics in book form in Germany until the last decade and only now are translations of the six volumes becoming available here. The present volume is Volume I of the series, although the order of publication of the translations has not followed the series order.

Mechanics, as the oldest branch of physics, has been studied and taught for so many years that its content and the subdivision of its material is almost rigidly defined. The author of an introductory text can hardly be expected to show individualism in much more than his emphases and his choice of examples. Within this framework, Sommerfeld's *Mechanics* proves to be a coherent and interesting text on the dynamics of particles and of rigid bodies. It is not excessively philosophic, but is still careful in its definitions, emphasizing fine points which are too often glossed over in elementary works. Its text is readable, and pleasantly enlivened by occasional brief historical references.

The treatment is comprehensive and well illustrated by examples which often have a strong intuitive appeal, in contrast to the rather abstract examples used in some other texts. The rigid body is particularly well treated, without recourse to the use of Euler's angles, and is illustrated by discussions of tops and gyroscopes in an expository fashion.

Variational principles and the Lagrangian and Hamiltonian formulations of mechanics are not introduced until the last three chapters. The application of Lagrange's equation is illustrated by a selection of examples treated by more elementary methods in the preceding portion of the text.

As in any modern text on the subject, vector methods are the heart of the mathematical analysis. However, little or no use is made of vector calculus, and the reader need only be familiar with vector algebra.

The translation is smooth and readable, and apparently was made from the fourth German edition, although the preface to that edition is not given, and the translation of the preface to the first edition is somewhat abbreviated. The figures have been reproduced from the German text by photographic means; the necessary retouching has been clumsy in spots. The equations have been wholly reset, since the notation was changed in translation. Some equations are printed in an uncomfortably small type face. At one place in the text the original German notation remains, but no real confusion should arise. P. P. Ewald has written an introductory note for the series which is printed in this volume.

The book should make an excellent text for use in advanced undergraduate and first year graduate courses. An alert student will sometimes demand more justification than is given in the text, but such discussion makes for a lively class. A. C. BYERS

KINETICS AND MECHANISM: A STUDY OF HOMOGENEOUS CHEMICAL REACTIONS, by A. A. Frost and R. G. Pearson. 343 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons; London, Chapman & Hall, Ltd.; 1953. Price, $6.00.

In adding to the already relatively long list of recent books on chemical kinetics, the authors wished to bridge the gap between a purely kinetic treatment of chemical reactions and a purely mechanistic one. Their aim was to show the intimate relationship between the kinetic law followed by a chemical reaction and its mechanism in terms of radicals, atoms, and ions. This is a daring enterprise, since although it is well known that the kinetic method is a powerful one in throwing light on the intricacies of a reaction path, all too often the answer is not unique, and must be supplemented by some other kind of factual information. The authors are well aware of this weakness of the method. The first part of the book is devoted to a discussion of general kinetic arguments such as the treatment of kinetic data and the experimental methods available for obtaining such data, kinetic theory of gases, absolute rate theory, comparison of theory with experiments, reactions in solution, complex reactions, homogeneous catalysis, and chain reactions. It is in this section that the treatment overlaps published books on chemical kinetics. In the second part a selected group of typical examples of reactions are given, where the authors show how the application of the kinetic treatment has been able to give an insight into the reaction mechanism. These examples are mostly taken from reactions in solutions, mainly because of the lack of gas phase reactions with clean cut kinetics and well established mechanisms. The writing is clear and the treatment is simple which makes the book most valuable for workers in the field or as a classroom texbook.

G. PARRAVANO

RECORD OF THE ROCKS, by Horace G. Richards. 413 pages, illustrations, 16 × 24 cm. New York, The Ronald Press Company, Inc., 1953. Price, $6.00.

Long familiar with the rocks of eastern North America—New York, New Jersey, Pennsylvania, Delaware, Maryland, Virginia and the Carolinas—and aware of the richness of the geological evidence, Mr. Richards, associate curator of Geology and Paleontology at the Academy of Natural Sciences in Philadelphia, has traced the development of the earth from earliest times to the present by concentrating on this section of the country. Written clearly and fluently, *Record of the Rocks* is unusually readable and because of its arrangement, should interest layman or student equally well. Mr. Richards has introduced his subject in a very gradual manner so that one need have had no previous specialized study in order to understand the material.

The description of how rocks are formed, how they are interpreted by the geologists and the geologic timetable constitutes the opening pages of the book. Following these introductory chapters, the movements of the land and sea, the definition and formation of fossils and the classification of animals and plants are discussed. Just before commencing with the details of the main divisions of geologic time, Mr. Richards has very interestingly included a resume of the three main theories of the origin of the earth: the nebular hypothesis, the planetesimal hypothesis and the gaseous hypothesis.

Having presented the background material, Mr. Richards then proceeds to the details of the various geological epochs beginning with the pre-Cambrian, and down through those of the

Paleozoic, Mesozoic and Cenozoic eras. About each of these epochs, the history and geological activity, the life and the economic resources are given. Numerous maps, charts, pictures and restorative plates of the life in the various times greatly enliven the book.

E. W. HAMMER

MICROMETEOROLOGY, by O. G. Sutton. 333 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1953. Price, $8.50.

Micrometeorology is a study of physical processes in the lowest layers of the earth's atmosphere, which does not yield its secrets willingly. Problems of the lower atmosphere are familiar in other branches of applied science although the approach is often different from that used in micrometeorology.

The techniques required to follow the discussions in the book involve a familiarity with college mathematics and physics. Manipulations involve partial differential equations, triple integrals, hyperbolic functions, error functions, and the Laplace operator, while developments are based upon a knowledge of classical physics and most of the principles are based upon fluid mechanics and thermodynamics. Chapters are devoted to laminar and turbulent flow, heat transfer, radiation, diffusion and evaporation. The basic physical laws are applied to the air close to the ground, and the reader is challenged to find specific application of the material presented. Although the book is intended to be used as a textbook, no illustrative problems are presented and no exercises are suggested; however, each chapter ends with an extensive bibliography which would be invaluable for further study or research.

An excellent companion to *Micrometeorology* is a book on microclimatology entitled *The Climate Near the Ground* by Dr. Rudolf Geiger, published in English by the Harvard University Press in 1950. This book reveals results from experiments on microclimates throughout the world. Over 800 references are employed, most of them in German. The same subjects are considered in both books. The former is highly analytical and an excellent contribution to micrometeorology. The latter is somewhat analytical and permits the solution of problems by analogy. Together they explain the behavior of the atmosphere near the ground.

A. DONALD HAY

COMPUTING METHODS AND THE PHASE PROBLEM IN X-RAY CRYSTAL ANALYSIS, edited by Ray Pepinsky. 390 pages, illustrations, diagrams, 21 × 28 cm. State College, The Pennsylvania State College, 1952. Price, $7.50 (bound in boards).

Computing Methods and the Phase Problem in X-Ray Crystal Analysis is a compilation of papers given at a conference held at the Pennsylvania State College April 6–8, 1950 to consider that topic. The phase problem is now, as it was at the time of the conference, the most perplexing facing the workers in the expanding field of X-ray crystallography. This book, being a collection of fundamental papers on the subject, is a convenient and worthy report on approaches to the problem.

A list of authors of the papers gives a good indication of the worth of the book. It is regrettable that a few of the papers given at the conference could not be included. But, while those of Ewald, Karle and Hauptman, and Zachariasen are missing, the impressive list of contributors includes Pepinsky, Patterson, Beevers, Robertson, Lipson, MacGillavry, Buerger, Bijvoet, Goedkoop, and a number of other prominent names in X-ray crystallography. Each contributor is, unquestionalby, an expert in the field to be covered—in many cases, the foremost authority.

The conference (and the book) opened with introductory papers by Pepinsky and Bijvoet. Then followed eighteen papers discussing methods of attacking the phase problem, for example, Patterson syntheses, equalities and inequalities, diffuse scattering, and optical methods, and methods of computation, for example, punched cards, digital computers, and analogue computers (including, of course, X-RAC). There is little that can be said about the theoretical papers which is not already known to workers in the field from reading articles in the various periodicals. The coverage is certainly extensive and informative.

The most important part of the book, however, is probably that on computers. And while there is much said about other methods of computation and the relative merits of digital

and analogue computors are discussed, it is X-RAC which is in the limelight. Pepinsky explained the beginnings of the idea, the construction of X-RAC with schematic diagrams and detailed explanation of its functioning, the mathematics of X-RAC, it uses, the preparation of the data, and the presentation and accuracy of results. He also discusses S-FAC, the structure factor computer. Some of these topics are further discussed in the nine appendices. Worthy of special note is Sayre's thesis work, done on the X-RAC program, on Fourier transforms. Anyone interested in using X-RAC who is not entirely familiar with it would profit considerably by reading this account.

The printed discussions are augmented by numerous diagrams and pictures. This is especially true of the description of X-RAC which includes diagrams and pictures of parts of the machine and of personnel connected with it and persons attending the conference.

M. H. BORTNER

TEXTBOOK OF ENGINEERING MATERIALS, by Melvin Nord. 518 pages, diagrams, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.50.

Dividing his book into four principal parts—raw materials, production, properties, and specific engineering materials—Dr. Nord, Professor of Chemical Engineering at Wayne University, has covered rather thoroughly the subject of engineering materials. In a field of such a magnitude as this, of course, each phase of the subject cannot be investigated completely; but the basic concepts behind each of the four parts into which the book has been divided are presented. Since the book is intended for college freshman and sophomore use, much more than this is not required.

The short first part of the book on the raw materials of engineering presents, as one would expect, the sources, formation and recovery of minerals. It is really some very elementary physical geology. In the second part, the production of engineering materials from the crude ores are discussed. Mineral dressing, such chemical processes as leaching, calcination, roasting, combustion and the branches of extractive metallurgy—pyrometallurgy, hydrometallurgy and electrometallurgy— are described. Beginning with the fundamental states of matter, the mechanical, physical and chemical properties of engineering materials are discussed in the third part. The fourth part, with which more than two-thirds of the book is concerned, deals with such specific materials as those for the generation of energy (water, fuels and explosives); ferrous and non-ferrous metals and alloys; natural and artificial building stones; natural and synthetic organic materials as wood, rubber and plastics; and protective materials as surface coatings and lubricants. Each of the various materials is described in some detail. In the case of the ferrous and non-ferrous materials, for instance, the source, production, properties and uses are discussed. In the case of cementing materials used in building stone construction, the various cements and the chemical reactions occurring during setting are discussed. In the case of rubber, the production, vulcanization, latex processing, the various synthetic rubbers, and the physical properties are discussed.

Whole books have been written about each of these materials, so that one should not expect to find everything available on each in this volume. But one will get an over-all survey of the broad field of engineering materials, what they are, where they come from and why and where they are used.

E. W. HAMMER

AN INTRODUCTION TO SCIENTIFIC RESEARCH, by E. Bright Wilson, Jr. 375 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $6.00.

Young men and women often complete scientific studies and enter fields of research without ever having been exposed to any formal training in methods of scientific research. This is most unfortunate because often this results in a slow rate of professional growth and contributes to waste and inefficiency in research as a whole. Dr. Wilson's book provides answers to many of the perplexing problems encountered by scientists, young and old. The thoughtful reading of this book will not guarantee success and efficiency to those who are beginning research careers in any branch of science, but it will most certainly improve them.

In lieu of attaching oneself to an eminently successful research scientist to learn techniques, one can read about procedures which successful investigators have found helpful. Not

confining himself to his particular field of physical chemistry, the author covers topics which are common to research as an activity.

Every piece of research starts with a problem. In the first of the thirteen chapters, Dr. Wilson discusses how to choose problems for research, how to evaluate them for potential fruitfulness, and how to conduct oneself with fellow scientists and colleagues. Every young scientist has these problems. Following a chapter on the importance of a literature survey, the author delves into the elements of the scientific method, wherein he discusses the roles of authority in science, hypotheses and their treatment. An excellent chapter on the principles of designing experiments is followed by one on designing apparatus. This reviewer is of the opinion that the latter is the one weak part of the book, where, in an attempt to draw examples of the construction of apparatus from various fields of endeavor, the author masters none. How to conduct experiments, how to analyze experimental data, with useful advice on mathematical techniques, and how to report the results of research are treated very successfully in the rest of the volume.

This fine book, while written originally for those who are beginning careers, is of equal value to those experimental researchers who have never had formal training in methods of research. It is also recommended for those who supervise, direct and sponsor technical research programs.

S. CHARP

STATISTICAL THEORY IN RESEARCH, by R. L. Anderson and T. A. Bancroft. 399 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $7.00.

This book provides the material for a year's college course in statistics and assumes that the student has a familiarity with calculus. It provides an introduction to statistics with sufficient mathematical detail to make the data-treating procedures developed logically satisfying to a student of engineering. Throughout, there are numerous illustrative examples as well as problem exercises.

The first half of the book treats basic statistical theory and has particularly good chapters on derived sampling distributions on tests of hypotheses. The second half of the book deals with the analysis of experimental models by least squares. The basis for a thorough understanding of the principles of experimental design is provided and many detailed examples of types of design models provide practical illustration of the principles.

All in all, this book should be valuable to the research engineer or scientist. In the past, sophistication in statistical methods has been limited, by and large, to students of the biological and social sciences. A text such as this makes it possible for engineers to attain competence in the use of statistical methods. Although no book can hope to meet every reader's tastes, I should like to make the minor criticism that the omission of any discussion of sequential analysis techniques is a shortcoming.

EZRA KRENDEL

SIR JAMES JEANS, by E. A. Milne. 176 pages, 15 × 23 cm. New York, Cambridge University Press, 1953. Price, $4.00.

Sir James Jeans died more than six years ago but we have had to wait until now for a biography. Even after the long wait we are not presented with a complete picture of the man, which is comprehensible, since he was a shy, retiring creature. The special merit of the book by the late E. A. Milne is its frank and penetrating study of Jeans as a scientist.

After a precocious childhood, Jeans went to Cambridge where his mathematical genius secured for him a valued fellowship. His earliest work was concerned with the distribution of velocity among the molecules of a gas. In following the development of his life's work, in which he became best known for his writing in the field of astronomy, one perceives the interweaving of two strands of interest in mathematical physics, the analysis of the very small (molecular physics) and the analysis of the very large (cosmogony). His work in the two distinct fields secured for him such a well earned reputation that, at the age of twenty-eight, he was invited by Woodrow Wilson to come to Princeton where he lectured to the advanced students. In the same year he was elected a Fellow of the Royal Society. Four years later he returned to England as Stokes Lecturer at Cambridge, and in 1912 he retired to devote himself to mathematical research. Within two years he had closed his work on atomic phe-

nomena or statistical mechanics with his justly famous *Report* on radiation and the quantum theory, which contributed to the acceptance of Planck's theory.

A new phase opened with his superb essay *Problems of Cosmogony and Stellar Dynamics* which, in spite of some revisions in the light of later discoveries, has remained a classic in astronomy. This was about the time when Eddington began to publish his researches on the equilibrium of the stars with which Jeans found fault and which led to the famous controversy between the pair. More important, Jeans was elected secretary of the Royal Society, and in this office he succeeded in securing for the Society's *Proceedings* the most important original papers in physics.

The final phase of Jeans' life was devoted to the popular exposition of the theories and discoveries of astronomy and physics, to which he brought a félicity of language, a clarity of exposition, an aptness of simile, and a width of knowledge which acquired for his books an enormous circulation.

The late E. A. Milne, himself a distinguished astronomer, was a happy choice to write the biography of Jeans. He was fully qualified to appreciate the intellectual achievements of Jeans and to assess his contribution to the science of his day. The book is written with understanding and penetration, a perception of shortcomings, and a full realization of what is significant in the work of a man who was not easy to understand, either as a scholar or as a man. It leaves little more to be said on the labors of the eminent English scientist. T. C.

THEORY OF ELECTRIC POLARISATION, by C. J. F. Böttcher. 492 pages, diagrams, 16 × 23 cm. Houston, Elsevier Press, 1952. Price, $10.00.

The field of dielectric studies has not been blessed with many good books. Debye's basic "Polar Molecules," C. P. Smyth's "Dielectric Constant and Molecular Structure," Van Vleck's "Theory of Electric and Magnetic Susceptibilities," Le Fevre's "Dipole Moments," and Frohlich's "Theory of Dielectrics" about complete the list of titles in English. Of these, only Frohlich's book is of recent origin, and it is rather limited in scope. The appearance of this very comprehensive book by Professor Böttcher should therefore be very welcome to teacher, student, and research worker alike in this field.

The first five chapters are devoted more or less to the development of fundamental electrostatic theorems. These theorems are used to explain the experimental data in the last six chapters. The general development of the electrostatic potential in series form is given, and is applied to numerous cavity and other boundary value problems. Reaction fields for spherical and ellipsoidal cavities containing dipoles and higher multipoles are calculated. The energy relationships in the interaction between matter and the field are developed quite thoroughly in the long fifth chapter.

The last half of the book treats the polarization occurring under various conditions, including low and high frequency fields, and in the optical region. Both polar and non-polar dielectrics are discussed. In the case of low frequency fields, they are given separate chapters, and another is devoted to the determination of permanent dipole moments. Four appendices deal in greater detail with Legendre functions and other mathematical apparatus.

The treatment is everywhere that of classical electrostatics, and represents the most complete such discussion in this particular field known to this reviewer. The central purpose of the book is the development of this classical theory. Although the experimental data are brought in as illustration for the theory, the book also represents an excellent review in this respect as well. The few sketchy spots in this review are almost all in places where the underlying theory is basically quantum mechanical in nature, or contains other elements of a non-electrostatic nature. Such places occur in the treatment of microwave absorption in gases, and of ferroelectrics.

All told, it is an excellent book, and should be recommended to all serious students in this field. ALAN D. FRANKLIN

FLYING SAUCERS, by Donald H. Menzel. 319 pages, illustrations, 14 × 21 cm. Cambridge, Harvard University Press, 1953. Price, $4.75.

Flying saucers have been with us for a long time—perhaps 3,000 years—but only today in this age of instantaneous communication can it really affect the entire population of a country.

In the late 18th Century flying saucers were seen by the French. In the mid-19th Century they were seen in England as the famous "Durham Lights." Today, as we pass the mid-point of the 20th Century, we, in the United States, see them. If we consult history a remarkable coincidence is uncovered. Every nation which was the leader in the world at that time was the nation subject to these apparitions. Psychologists might tell us that people who have most to lose are those most apt to see a burglar in the shadows. Could this refer to nations, too?

Now, in answer to these questions, there appears on the scene a book explaining the formation, appearance and behavior of these exciting objects. This book is no accident. It is the natural evolution of some magazine articles and stories written by the author which appeared during 1952.

In *Flying Saucers*, Dr. Donald H. Menzel, Acting Director of the Harvard College Observatory, and one of the world's ranking astrophysicists, describes in fascinating detail the formation of the ubiquitous saucers. He shows how these can appear as a result of atmospheric phenomena. He explains the role that mist, ice crystals and mirages play in the creation of these apparitions for these appear to be the prime source of flying saucer formations.

In detail he describes some of the more spectacular of the sightings. He recounts the experience of Captain Thomas F. Mantell who crashed to his death in a P-51 chasing a flying saucer in Kentucky and shows it could have been a "mock sun" caused by ice crystals in high cirrus clouds.

The "little men from Venus" are fully treated and this brings up a case in point. Dr. Menzel shows how the story originated at the University of Denver in a lecture class and how the story spread like wild fire across the front pages of this country. Yet, when J. P. Cohn of the *San Francisco Chronicle* cracked the story and proved it was a hoax and was to lay a groundwork for a future stock promotion scheme (the mysterious scientist X and Dr. Gee are under indictment for fraudulent stock practices) the newspapers of this country either buried this information on page 29 or omitted it as not being newsworthy.

During some lectures this reviewer has given on flying saucers by actual count about 40 per cent of the audience signified they heard of the "little men from Venus" yet less than 1 per cent actually knew the story which Dr. Menzel relates in his book.

The remarkable jousting of Lt. George F. Gorman with a flying saucer is vividly described and the explanation of this dogfight is given. Throughout the entire book are spread case histories of flying saucers seen from every conceivable position and under every conceivable circumstance.

Flying Saucers is welcome as the first sober volume in the growing literature on a subject which has confused even some educated persons. It will allay the anxieties and apprehensions to which many find themselves subject to those who insist on flying saucers being interplanetary in origin it will permit the adoption of a saner attitude. Dr. Menzel has succeeded with this book in placing himself at the head of flying saucer debunkers. He is to be congratulated on a magnificently written book which was sorely needed by a throng of troubled souls.

I. M. LEVITT

BOOK NOTES

THE MATHEMATICAL THEORY OF NON-UNIFORM GASES, by Sydney Chapman and T. G. Cowling. Second edition, 431 pages, diagrams, 19 × 27 cm. New York, Cambridge University Press, 1953. Price: $10.50 (complete); $1.00 (new material).

One possible solution to the problem of keeping technical book up-to-date at a reasonable cost is offered by Cambridge University Press in a new second edition of this 1939 volume. Their solution has been to issue the new material as a separate work. This slender volume is the same size as the first edition, making it convenient for keeping the two side by side on a bookshelf. Those who have the first edition may purchase the new material—entitled "Notes Added in 1951"—for $1.00; new readers can purchase the second edition, which has the new material bound in with the old, for $10.50. Libraries and scientific readers will welcome this new, economical approach to the problem of keeping the latest material available at the lowest cost. While not all books lend themselves to this method of revision, surely enough could be done this way to effect a tremendous savings.

CODE FOR PROTECTION AGAINST LIGHTNING, prepared by the National Bureau of Standards in cooperation with the National Fire Protection Association and the American Institute of Electrical Engineers. 88 pages, diagrams, plates, 13 × 19 cm. Washington, Government Printing Office, 1953. Price, $0.40.

In order to enable the public to demand adequate lightning protection, the National Bureau of Standards has just published this *Code*, NBS Handbook 46. The pamphlet includes material revised from previous handbooks, as well as new rules and specifications for various types of lightning protection. Study of this *Code* should result in more effective protection, and it is, therefore, recommended to all owners of property, including buildings, vessels, livestock, grain elevators, radio antennas, wire fences and storage tanks for flammable liquids and gases. In addition to information on lightning protection, the booklet has sections on lightning and its effects, the character of the damage it can cause, and thunderstorm data. A most useful contribution.

REVIEW OF TEXTILE PROGRESS, VOLUME 3, edited by W. J. Hall and others. 590 pages, 14 × 21 cm. Manchester, The Textile Institute, 1952. Price, 35/; 28/ (members).

Volume 3 of this yearly review is now available, covering developments in the textile industry during 1951. In eleven sections, 31 papers are presented, thoroughly detailing progress in each of the subjects. Additions to the present volume include papers on the physics and chemistry of nylon, on sizing and sizing materials, on felt, and on industrial applications of textiles. While the range of subjects is very wide, the authors of the individual papers have included a great amount of detail. The volume, therefore, will be welcomed by textile students and technologists alike, as an authoritative record of textile progress.

DAIRY ENGINEERING, by Arthur W. Farrall. Second edition, 477 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $6.00.

Improvements in dairy equipment and dairy-plant design, since 1942 when the first edition was published, make it desirable to revise this text. Emphasis is still on the engineering phases of the dairy industry, with the text aimed at students as well as operators. New material has been added on plant design, waste disposal, labor-saving methods and servicing of dairy equipment. The problems of sanitary standards and handling of diary products are also discussed. With the addition of this new material, the volume should prove indispensable to dairy engineers.

STATISTICAL THERMODYNAMICS, by Erwin Schrodinger. Second edition, 95 pages, 13 × 19 cm. New York, Cambridge University Press, 1952. Price, $1.75.

The only change over the first edition is the addition of a 7-page appendix, containing proof that a consistent procedure, based on very simple assumptions, always gives the same results. This appendix was added to offset the view—questioned in Chapter II, but used throughout the treatise—that a physical process consists of continual jump-like transfers of energy parcels between microsystems.

CURRENT TOPICS

Unique Mine-Ventilation System.—The only one of its type in Canada, a towering 20-ton exhaust fan, shaped like a huge malted milk container, has been installed by The International Nickel Company of Canada, Limited, to ventilate the underground workings of the new caving project at its Creighton Mine in the Sudbury District of Ontario.

The giant vertical fan, driven by a 350-hp. motor weighing five tons, ventilates the workings by a flow of fresh air at the rate of 300,000 cu. ft. per minute. The fan draws the air down from the surface, directly through the caved or broken ore, circulates it through the underground workings and carries it back to the surface through a main return shaft. It is at the top of this shaft that the fan is mounted.

The fan stands 42 ft. above its concrete base, and the diameter at the inlet is 15 ft. The 124-in. impeller has 12 stainless steel blades which are adjustable in pitch through 25 degrees to accommodate the load as the mining operation moves farther from the main return air shaft.

In the caving method of mining at Inco's properties in the Sudbury area, gigantic masses of undercut ore, far underground, are induced to cave and disintegrate through tension and torsion as well as of their own weight. This low-cost bulk mining method, plus metallurgical practices, makes it practicable for International Nickel to recover and treat ore lower in grade than it has ever worked in underground mining. The supply of economically available ore has thus been broadened, enabling the Company to maintain its current nickel production at a post-war high. The bulk mining methods are part of Inco's $150,000,000 major long-range program of conversion to wholly underground mining.

As the Creighton caving program took shape on the drafting boards, primary consideration was given to devising a ventilation system which would provide for the flow of a steady stream of fresh air through the slusher drifts, passageways through which ore is drawn off after it has come through funnel-shaped boxholes beneath the mass of broken ore.

The unique ventilation system is effectively servicing the caving project as it has been developed to date, and has ample capacity to handle ventilation needs under maximum operating conditions.

Industrial Psychiatry Increases Engineering Efficiency.—Helping engineers straighten out their emotional problems is as important to effective industrial output as providing them with the latest tools and equipment, according to a feature article in the February *American Engineer*, the official magazine of the National Society of Professional Engineers.

Called "The Human Factor in Engineering Work," the story tells how the du Pont Company is making use of industrial psychiatry to increase the efficiency of its engineering staff. Dr. F. W. Dershimer, in charge of du Pont's industrial pyschiatry program, pointed out that running away from problems is the biggest single cause of nervous breakdowns and emotional and mental

difficulties that are reflected in poor performance on the job. Facing and solving problems is the only way to handle them, he said.

He also cited the case of a research group of 160 du Pont employees who were given deadlines for their work assignments—something not usually done with people engaged in creative and research activities—and pointed out that their production was practically doubled. "I think it is attributable to straightening out the mental processes of the scientists and engineers concerned," Dr. Dershimer said. "When all emphasis was put on performance, they discarded the extraneous and concentrated on the most promising leads."

In dealing with employees who are troubled by emotional disturbances or problems, sympathy is desirable, Dr. Dershimer said, but "it should not be translated into endless coddling and appeasement. . . . Industry through management and supervision can best help the emotionally sick employee by requiring normal performance from him. Finding ways to get such individuals to live up to their normal responsibilities is the best thing that can happen to them. Mental health and well-being improves with responsible behavior."

While industrial psychiatry is a relatively new approach to the problems of our industrial system, Dr. Dershimer also emphasized the remarkable results already attained in safety and accident prevention work. Helping so-called "accident prone" workers meet their emotional problems effectively can actually reduce the rate of industrial accidents, of which more than 80 per cent can be traced directly to human rather than mechanical failures.

"The increased strain put on many engineers because the shortage of engineers requires increased individual production with engineering staffs often spread dangerously thin, makes it natural that mental hygiene will play an important role in the utilization of engineers," the article concludes.

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AEC Survey Shows Growth of Radiation Instrument Industry.—The radiation instrument industry, virtually non-existent in 1946, had an annual business volume of approximately $20,000,000 and employed more than 2,400 persons in 1952, according to a survey conducted by the U. S. Atomic Energy Commission.

Growth of the new industry has paralleled development of the nation's atomic energy program since early 1947, when the AEC adopted a policy of encouraging its operating contractors to procure radiation instruments from commercial manufacturers. Before that time, security restrictions had required contractors to the Manhattan Engineer District, predecessor to the AEC, to design, develop, and produce their own radiation instruments.

The survey shows an expanding market for radiation instruments outside of the AEC program as well as within AEC installations. According to the survey, military agencies of the Government now provide about 50 per cent of the total market; the AEC and its principal contractors provide about 30 per cent, and the remainder is accounted for by private industry, universities, hospitals and research institutes, civil defense, export, and uranium ore prospecting.

More than 50 patents in the field of radiation instruments are owned by the U. S. Government and held by the AEC. A total of 51 non-exclusive, royalty-free licenses have been granted on these patents. Application for such licenses may be made to the Chief, Patent Branch, Office of the General Counsel, U. S. Atomic Energy Commission, Washington, D. C.

Liquid-Cooled Generator.—The General Electric Company will manufacture the first large generator with liquid-cooled conductors in the history of the electrical industry, it was announced recently by Glenn B. Warren, Vice President of the Company and General Manager of the Turbine Division.

A new method of circulating a liquid through hollow conductors will be used in the stator of a turbine-generator set for the new Eastlake power plant of the Cleveland Electric Illuminating Company.

The unit, consisting of a tandem-compound turbine rated at 208,000 kilowatts and a generator rated at 260,000 kilovolt-amperes, will be one of the largest in the world. This generator, alone, will be capable of supplying the household electrical needs of 600,000 people.

In making the announcement, Mr. Warren said that this type of liquid cooling makes possible a significant increase in capability of generators without increasing physical dimensions of the units. The higher capability results from more efficient removal of heat produced during the generation of electricity.

The present method of cooling large generators is to circulate hydrogen gas through passages in the magnetic portions of the rotor and stator. Developed by G. E., this method is highly efficient for units up to approximately 230,000 kilovolt-amperes capability at 3600 rpm.

The liquid cooling method makes possible the design and manufacture of generator armatures having much higher capability than present units.

Other technical features of the Cleveland machine include direct cooling of the rotating field winding with hydrogen, a new and improved grain-oriented strip steel in the magnetic portion of the armature, and an improved type of insulation.

G. E. is also supplying three conventional hydrogen-cooled units, each of 125,000 kilowatts capacity, for the Eastlake plant, which will go into operation this summer. The 208,000 kilowatt generator is scheduled for installation by the end of 1955.

Alphatron* Vacuum Gauge.—Answering the long-recognized need for a dependable instrument to measure absolute pressures from atmospheric down to 0.0001 mm; type 511 Alphatron (R) Vacuum Gauge has been developed by the National Research Corporation, Cambridge, Mass.

This new unit is a refinement of earlier models. The principle of operation is related to the conventional hot filament ionization gauge. The important difference is that the Alphatron gauge employs a shielded radioactive source instead of a hot filament. Alpha particles ionize the gas. Ionization current produced is measured by a d-c. amplifier which is calibrated to give absolute pressure in millimeters of mercury.

The sensing element and first stage amplifier are housed in a steel envelope which weights only four ounces. Attachment to the vacuum system is made by a $\frac{1}{8}$ in. standard pipe thread tubulation. The power supply, high stage amplifier, range selector, and output meter are incorporated in a compact stainless steel housing weighing seven pounds. Dependence on outside power supply which is subject to fluctuation is eliminated by the use of readily avail-

*Registered Trade-mark.

able batteries. The use of batteries eliminates an expensive regulated direct current power supply and substantially reduces the cost of the gauge.

Among the improvements which have been incorporated in the new unit is a greatly increased range. Linear indications of total pressure are provided between 1000 millimeters and 10^{-4} millimeters by ranges as follows:

Range	Smallest Division
0–1000 millimeters	10 millimeters
0–100 millimeters	1 millimeters
0–10 millimeters	.1 millimeters
0–1 millimeters	.01 millimeters
0–0.1 millimeters	.001 millimeters
0–0.01 millimeters	.0001 millimeters

Accuracy is plus or minus 2 per cent of full scale. Speed of response to pressure changes is instantaneous making the gauge useful also as a leak detector. The Alphatron ionization chamber is far less sensitive to contamination than hot filament or thermo-conductivity types of gauge chambers. When heavy contamination resulting from unusual operating conditions does occur, cleaning is accomplished simply by flushing with a suitable solvent.

Type 511 Alphatron gauge is particularly useful on vacuum coating units, furnaces, stills, drying and impregnating units, test chambers, and pumping systems. It is equally versatile as a laboratory instrument.

New Gauge Developed for Wind Tunnel.—A new form of basic instrumentation for the precise measurement of gas pressures in the range between absolute vacuum and one atmosphere has been developed at the Navy Ordnance Laboratory. The *new gauge* consists of two small beryllium-copper dish-shaped diaphragms which have been fastened to opposite sides of a ring to form a vacuum tight chamber. Two sizes of this precision pressure gauge are available at present. One is $\frac{3}{4}$ inch in diameter and $\frac{3}{8}$ inch thick and can measure a change of pressure as low as 0.01 millimeter of mercury. The other gauge is $\frac{9}{16}$ inch thick and can measure a change of pressure as low as 0.05 millimeter of mercury.

Hundred-year Tests on Strength of Concrete.—Well-cured concretes and mortars, like wine or cheese, definitely improve with age. Now in their 43rd year in the University of Wisconsin College of Engineering, hundred-year tests on the strength of concrete have so far revealed that some concrete and mortar mixes, unlike many other material things, increase considerably in strength with age. Under present plans, the long-time tests on the curing and aging strengths of concretes and mortars on Wisconsin's engineering campus will continue until the year 2023.

Among the first to be initiated in America, these Wisconsin tests are credited with giving engineers early reliable information. The tests show that over the first 43 years, concrete stored outside, where it is subject to all kinds of Wisconsin weather, becomes more than three times stronger than it was during the first days of its hardening away back before World War I. The Wisconsin tests are watched with great interest by construction firms of all kinds—road, bridge, and building—since they are of considerable value in both public and private construction involving use of concrete or mortar.

Dean Morton O. Withey of the College of Engineering inaugurated the tests at UW in 1910, five years after he came to the University in 1905 as an instructor in mechanics and in the Materials Testing Laboratory. Reaching the automatic retirement age of 70 at the end of this year, Dean Withey brings to a close nearly half a century of service in Wisconsin engineering education and research.

For the curing and aging strength tests, Dean Withey explains that the concrete mixes were made into cylinders 6 in. in diameter and 12 in. high for compression tests. The mortars were made into 2-in. cubes for compression tests and briquets for tensile or "pulling" tests. Three series of tests were begun, one in each of the years, 1910, 1923, and 1937.

Different mixes were used such as a "neat" mix, 1:1 and 1:3, and 1:2:4 1:3:6 mixes. Translated, the "neat" mix consists of pure cement and water; the 1:1 mortar mix of 1 part cement to 1 part sand; the 1:3 mix containing proportions in that order; the 1:2:4 concrete mix consists of 1 part cement to 2 parts sand and 4 parts crushed stone, with the 1:3:6 mix containing proportions in that order.

The products have been curing and aging under these conditions: outdoors in a cellar, and indoors in water.

Under the tests, the compressive strengths of concrete increased when cured unprotected outdoors in the Wisconsin climate; while cured indoors at lower humidities the strengths decreased after three months.

Although the outdoor exposure was not the most severe, the data from the tests so far completed show that the concretes and mortars of usual proportions had very good weathering qualities.

It is estimated that during the first 30-year exposure, specimens of the 1910 series suffered 750 cycles of freezing and thawing. Under these conditions the strengths of the 1:2:4 specimens increased from 1,800 psi. at one month to 5,955 psi. pressure at 30 years and the strength of the 1:3:6 mix increased in nearly the same ratio.

"Shaking" the concrete well during placing by means of an internal vibrator reduced the water content, which probably improved both strength and volume constancy in the 1937 series of tests. The tests also revealed that the tensile or "pulling" strength of mortar suffers more from outside exposure than does the compressive or "squeezing" strength; and that the contractions and losses in weight of the specimens aged and cured indoors were about double those losses for specimens of the same age cured outdoors.

Dean Withey's tests are now widely credited with giving the engineering profession reliable information about concrete and mortar strengths years years after placement. This information is of great importance in estimating maintenance, depreciation, and replacement costs over the years in all kinds of concrete and mortar construction work. Dean Withey is the author of several score bulletins, papers, and reports on his studies in masonry material.